TITLE 126
LEGISLATIVE RULE
BOARD OF EDUCATION

SERIES 44BB
WEST VIRGINIA COLLEGE- AND CAREER-READINESS STANDARDS FOR MATHEMATICS (2520.2B)

§126-44BB-1. General.

1.1. Scope. – W. Va. 126CSR42, West Virginia Board of Education(hereinafter WVBE) Policy 2510, Assuring the Quality of Education: Regulations for Education Programs (hereinafter Policy 2510) provides a definition of a delivery system for, and an assessment and accountability system for, a thorough and efficient education for West Virginia public school students. Policy 2520.2B defines the content standards for mathematics as required by Policy 2510.

1.2. Authority. – W. Va. Constitution, Article XII, §2, W. Va. Code §18-2-5 and §18-9A-22.

1.3. Filing Date. – December 18, 2015.

1.4. Effective Date. – July 1, 2016.

1.5. Repeal of former rule. – This legislative rule repeals and replaces W. Va. 126CSR44BB WVBE Policy 2520.2B "Next Generation Content Standards and Objectives for Mathematics in West Virginia Schools" filed September 12, 2014 and effective October 14, 2014.

§126-44BB-2. Purpose.

2.1. This policy defines the content standards for the program of study required by Policy 2510 in mathematics.

§126-44BB-3. Incorporation by Reference.

3.1. A copy of the West Virginia College- and Career-Readiness Standards for Mathematics is attached and incorporated by reference into this policy. Copies may be obtained in the office of the Secretary of State and in the West Virginia Department of Education (hereinafter WVDE), Office of Middle/Secondary Learning.

§126-44BB-4. Summary of the Content Standards.

4.1. The WVBE has the responsibility of establishing high quality standards pertaining to all education standards pertaining to all education programs (W. Va. Code §18-9A-22). The content standards provide a focus for teachers to teach and students to learn those skills and competencies essential for future success in the workplace and further education. The document includes content standards that reflect a rigorous and challenging curriculum for mathematics.

§126-44BB-5. Severability.

5.1. If any provision of this rule or the application thereof to any person or circumstances is held invalid, such invalidity shall not affect other provisions or applications of this rule.

Introduction

West Virginia's College- and Career-Readiness Standards have been developed with the goal of preparing students in a wide range of high-quality post-secondary opportunities. Specifically, college- and career-readiness refers to the knowledge, skills, and dispositions needed to be successful in higher education and/or training that lead to gainful employment. The West Virginia College- and Career-Readiness Standards establish a set of knowledge and skills that all individuals need to transition into higher education or the workplace, as both realms share many expectations. All students throughout their educational experience, should develop a full understanding of the career opportunities available, the education necessary to be successful in their chosen pathway, and a plan to attain their goals.

West Virginia's College- and Career-Readiness Standards for Mathematics are the culmination of an extended, broad-based effort to help ensure that all students are college- and career-ready upon completion of high school. The skills contained in the mathematics standards are essential for college- and career-readiness in a twenty-first-century, globally competitive society. The standards reflect a progression and key ideas determining how knowledge is organized and generated within the content area. Standards evolve from specifics to deeper structures inherent in the discipline. These deeper structures serve to connect the specifics. The standards follow such a design, stressing conceptual understanding of key ideas and continually returning to organizing principles such as place value or the properties of operations to structure those ideas. The sequence of topics and performances outlined in mathematics standards must respect the scientific research about how students learn and what is known about how their mathematical knowledge, skill, and understanding develop over time.

The West Virginia College- and Career-Readiness Standards are the result of a statewide public review of the state's educational standards. The West Virginia Department of Education (WVDE), West Virginia Board of Education (WVBE), and West Virginia University partnered in this initiative that began with a website, Academic Spotlight, which served as the platform for feedback collection. This website was active July through September of 2015. After the comment period closed, comments were evaluated by a team of diverse stakeholders, who made recommendations to WVBE based on the comments to meet the needs of West Virginia students. Additionally, during the month of September 2015, eight universities around the state hosted town hall meetings where citizens could pose questions about the standards to a panel of teachers, administrators, and representatives from higher education. The West Virginia College- and Career-Readiness Standards reflect the improvements brought to light by these two methods of public input.

Explanation of Terms

Clusters are groups of standards that define the expectations students must demonstrate to be college- and career-ready.

Domains are broad components that make up a content area. Domains in mathematics vary by grade-level and by course. For example, the five domains for mathematics in Grade 6 are Ratios and Proportional Reasoning, The Number System, Expressions and Equations, Geometry, and Statistics and Probability.

Standards are expectations for what students should know, understand and be able to do; standards represent educational goals.

Number of Standards

The number for each standards is composed of three parts, each part separated by a period:

- the content code (e.g., M for Mathematics),
- the grade level or course, and
- the standard.

Illustration: M.3.1 refers to mathematics, grade 3, standard 1; M.C.1 refers to mathematics, calculus, standard 1.

Mathematics Grade Level or Course:

K	Kindergarten
1	Grade 1
2	Grade 2
3	Grade 3
4	Grade 4
5	Grade 5
6	Grade 6
7	Grade 7
8	Grade 8
1HS8	8^{th} Grade High School Mathematics I
1HS	High School Mathematics I
2HS	High School Mathematics II
3HS (without +)	High School Mathematics III LA
3HS (+)	High School Mathematics III STEM
3HS (*)	High School Mathematics III Technical Readiness
3HS (^)	High School Mathematics IV Technical Readiness
A18	8^{th} Grade High School Algebra I
A1HS	High School Algebra I
GHS	High School Geometry
A2HS	High School Algebra II
AMM	Advanced Mathematical Modeling
C	Calculus
4HSTP	High School Mathematics IV – Trigonometry/Pre-calculus
SRM	STEM Readiness
TMS	Transition Mathematics for Seniors

MATHEMATICS

The West Virginia College- and Career-Readiness Standards for Mathematics define what students should understand and be able to do in their study of mathematics. Asking a student to understand something means asking a teacher to assess whether the student has understood it. What does mathematical understanding look like? One hallmark of mathematical understanding is the ability to justify, in a way appropriate to the student's mathematical maturity, why a particular mathematical statement is true or where a mathematical rule comes from. There is a world of difference between a student who can summon a mnemonic device to expand a product such as (a + b)(x + y) and a student who can explain where the mnemonic comes from. The student who can explain the rule understands the mathematics, and may have a better chance to succeed at a less familiar task such as expanding (a + b + c)(x + y). Mathematical understanding and procedural skill are equally important, and both are assessable using mathematical tasks of sufficient richness.
The Standards begin with eight Mathematical Habits of Mind.

Mathematics: Mathematical Habits of Mind

The Mathematical Habits of Mind (hereinafter MHM) describe varieties of expertise that mathematics educators at all levels should develop in their students.

MHM1. Make sense of problems and persevere in solving them.
Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables and graphs or draw diagrams of important features and relationships, graph data and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method and they continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

MHM2. Reason abstractly and quantitatively.
Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to decontextualize—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to contextualize - to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand, considering the units involved, attending to the meaning of quantities, not just how to compute them, and knowing and flexibly using different properties of operations and objects.

MHM3. Construct viable arguments and critique the reasoning of others.
Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense and ask useful questions to clarify or improve the arguments.

MHM4. Model with mathematics.
Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

MHM5. Use appropriate tools strategically.
Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

MHM6. Attend to precision.
Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about

specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

MHM7. Look for and make use of structure.
Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7×8 equals the well-remembered $7 \times 5 + 7 \times 3$, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2×7 and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 – 3(x – y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.

MHM8. Look for and express regularity in repeated reasoning.
Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation $(y – 2)/(x – 1) = 3$. Noticing the regularity in the way terms cancel when expanding $(x – 1)(x + 1)$, $(x – 1)(x^2 + x + 1)$ and $(x – 1)(x^3 + x2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

Connecting the Mathematical Habits of Mind to the Standards for Mathematical Content

The Mathematical Habits of Mind describe ways in which developing students of mathematics increasingly engage with the subject matter as they grow in mathematical maturity and expertise throughout the elementary, middle and high school years. Designers of curricula, assessments and professional development should all attend to the need to connect the mathematical habits of mind to mathematical content in mathematics instruction.

Mathematics – Kindergarten

All West Virginia teachers are responsible for classroom instruction that integrates content standards and mathematical habits of mind. Students in kindergarten will focus on two critical areas: (1) representing and comparing whole numbers, initially with sets of objects; (2) describing shapes and space. Mathematical habits of mind, which should be integrated in these content areas, include: making sense of problems and persevering in solving them, reasoning abstractly and quantitatively; constructing viable arguments and critiquing the reasoning of others; modeling with mathematics; using appropriate tools strategically; attending to precision, looking for and making use of structure; and looking for and expressing regularity in repeated reasoning. The skill progressions begin in kindergarten as foundational understanding of numeracy. The following chart represents the mathematical understandings that will be developed in kindergarten:

Counting and Cardinality	Operations and Algebraic Thinking
• Count objects to tell how many there are by ones and by tens. • Write numbers from 0 to 20. • Compare two groups of objects to tell which group, if either, has more; compare two written numbers to tell which is greater. • Group pennies.	• Understand addition as putting together and adding to. • Understand subtraction as taking apart and taking from. • Add and subtract very small numbers quickly and accurately (e.g., 3 + 1).
Number and Operations in Base Ten	**Measurement and Data**
• Act out addition and subtraction word problems and draw diagrams to represent them. • Add with a sum of 10 or less; subtract from a number 10 or less; and solve addition and subtraction word problems. • Group objects by tens and ones. (1 group of 10 and 3 ones makes 13)	• Describe and compare objects as longer, shorter, larger, smaller, etc. • Classify objects and count the number of objects in each category. (e.g., Identify coins and sort them into groups of 5s or 10s.)
Geometry	
• Name shapes correctly regardless of orientation or size (e.g., a square oriented as a "diamond" is still a square).	

Numbering of Standards

The following Mathematics Standards will be numbered continuously. The following ranges relate to the clusters found within Mathematics:

Counting and Cardinality	
Know number names and the count sequence.	Standards 1-3
Count to tell the number of objects.	Standards 4-5
Compare numbers.	Standards 6-7
Operations and Algebraic Thinking	
Understand addition as putting together and adding to, and understand subtraction as taking	Standard 8-12

apart and taking from.	
Number and Operations in Base Ten	
Work with numbers 11-19 to gain foundations for place value.	Standard 13
Measurement and Data	
Describe and compare measurable attributes.	Standards 14-15
Classify objects and count the number of objects in each category.	Standard 16
Geometry	
Identify and describe shapes (squares, circles, triangles, rectangles, hexagons, cubes, cones, cylinders, and spheres)	Standards 17-19
Analyze, compare, create, and compose shapes	Standards 20-22

Counting and Cardinality

Cluster	Know number names and the count sequence.
M.K.1	Count to 100 by ones and by tens.
M.K.2	Count forward beginning from a given number within the known sequence (instead of having to begin at 1).
M.K.3	Write numbers from 0 to 20. Represent a number of objects with a written numeral 0-20 (with 0 representing a count of no objects).
Cluster	**Count to tell the number of objects.**
M.K.4	Understand the relationship between numbers and quantities; connect counting to cardinality. a. When counting objects, say the number names in the standard order, pairing each object with one and only one number name and each number name with one and only one object. b. Understand that the last number name said tells the number of objects counted and the number of objects is the same regardless of their arrangement or the order in which they were counted. c. Understand that each successive number name refers to a quantity that is one larger.
M.K.5	Count to answer questions (e.g., “How many?”) about as many as 20 things arranged in a line, a rectangular array, a circle, or as many as 10 things in a scattered configuration; given a number from 1–20, count out that many objects.
Cluster	**Compare numbers.**
M.K.6	Identify whether the number of objects in one group is greater than, less than, or equal to the number of objects in another group (e.g., by using matching and counting strategies).
M.K.7	Compare two numbers between 1 and 10 presented as written numerals.

Operations and Algebraic Thinking

Cluster	Understand addition as putting together and adding to, and understand subtraction as taking apart and taking from.

M.K.8	Represent addition and subtraction with objects, fingers, mental images, drawings, sounds (e.g., claps), and acting out situations, verbal explanations, expressions, or equations.
M.K.9	Solve addition and subtraction word problems and add and subtract within 10 by using objects or drawings to represent the problem.
M.K.10	Decompose numbers less than or equal to 10 into pairs in more than one way by using objects or drawings, and record each decomposition by a drawing or equation (e.g., 5 = 2 + 3 and 5 = 4 + 1).
M.K.11	For any number from 1 to 9, find the number that makes 10 when added to the given number by using objects or drawings, and record the answer with a drawing or equation.
M.K.12	Fluently add and subtract within 5.

Number and Operations in Base Ten

Cluster	**Work with numbers 11-19 to gain foundations for place value.**
M.K.13	Compose and decompose numbers from 11 to 19 into ten ones and some further ones by using objects or drawings, and record each composition or decomposition by a drawing or equation (e.g., 18 = 10 + 8); understand that these numbers are composed of ten ones (one ten) and one, two, three, four, five, six, seven, eight, or nine ones.

Measurement and Data

Cluster	**Describe and compare measurable attributes.**
M.K.14	Describe measurable attributes of objects, such as length or weight and describe several measurable attributes of a single object.
M.K.15	Directly compare two objects with a measurable attribute in common, to see which object has “more of” or “less of” the attribute, and describe the difference.
Cluster	**Classify objects and count the number of objects in each category.**
M.K.16	Classify objects into given categories, count the numbers of objects in each category, and sort the categories by count. Category counts should be limited to less than or equal to 10. (e.g., Identify coins and sort them into groups of 5s or 10s.)

Geometry

Cluster	**Identify and describe shapes (squares, circles, triangles, rectangles, hexagons, cubes, cones, cylinders, and spheres).**
M.K.17	Describe objects in the environment using names of shapes and describe the relative positions of these objects using terms such as above, below, beside, in front of, behind and next to.
M.K.18	Correctly name shapes regardless of their orientations or overall size.
M.K.19	Through the use of real-life objects, identify shapes as two-dimensional (lying in a plane, "flat") or three-dimensional ("solid").
Cluster	**Analyze, compare, create and compose shapes.**
M.K.20	Analyze and compare two- and three-dimensional shapes, in different sizes and orientations, using informal language to describe their similarities, differences, parts (e.g., number of sides and vertices/“corners”), and other attributes (e.g., having sides of equal length). Instructional Note: Student focus should include real-world shapes.

M.K.21	Model shapes in the world by building shapes from components (e.g., sticks and clay balls) and drawing shapes.
M.K.22	Compose simple shapes to form larger shapes (e.g., “Can these two triangles, with full sides touching, join to make a rectangle?”).

Mathematics – Grade 1

All West Virginia teachers are responsible for classroom instruction that integrates content standards and mathematical habits of mind. Students in the first grade will focus on four critical areas: (1) developing understanding of addition, subtraction, and strategies for addition and subtraction within 20; (2) developing understanding of whole number relationships and place value, including grouping in tens and ones; (3) developing understanding of linear measurement and measuring lengths as repeating length units; and (4) reasoning about attributes of, and composing and decomposing geometric shapes. Mathematical habits of mind, which should be integrated in these content areas, include: making sense of problems and persevering in solving them, reasoning abstractly and quantitatively; constructing viable arguments and critiquing the reasoning of others; modeling with mathematics; using appropriate tools strategically; attending to precision, looking for and making use of structure; and looking for and expressing regularity in repeated reasoning. Continuing the skill progressions from kindergarten, the following chart represents the mathematical understandings that will be developed in first grade:

Operations and Algebraic Thinking	Number and Operations in Base Ten
• Solve addition and subtraction word problems in situations of adding to, taking from, putting together, taking apart, and comparing (e.g., a taking from situation would be: “Five apples were on the table. I ate some apples. Then there were three apples. How many apples did I eat?”). • Add fluently with a sum of 10 or less, and accurately subtract from a number 10 or less (e.g., 2 + 5, 7 – 5). • Understanding the relationship between addition and subtraction.	• Understand what the digits mean in two-digit numbers (place value). • Use understanding of place value and properties of operations to add and subtract (e.g., 38 + 5, 29 + 20, 64 + 27, 80 – 50). • Identify the value of pennies, nickels and dimes.
Measurement and Data	**Geometry**
• Measure lengths of objects by using a shorter object as a unit of length. • Tell and write time.	• Make composite shapes by joining shapes together, and dividing circles and rectangles into halves or fourths.

Numbering of Standards

The following Mathematics Standards will be numbered continuously. The following ranges relate to the clusters found within Mathematics:

Operations and Algebraic Thinking	
Represent and solve problems involving addition and subtraction.	Standards 1-2
Understand and apply properties of operations and the relationship between addition and subtraction.	Standards 3-4
Add and subtract within 20.	Standards 5-6
Work with addition and subtraction equations.	Standard 7-8
Number and Operations in Base Ten	

Extend the counting sequence.	Standard 9
Understand place value.	Standards 10-11
Use place value understanding and properties of operations to add and subtract.	Standards 12-14
Measurement and Data	
Measure lengths indirectly and by iterating length units.	Standards 15-16
Tell and write time.	Standard 17
Represent and interpret data.	Standard 18
Geometry	
Reason with shapes and their attributes.	Standards 19-21

Operations and Algebraic Thinking

Cluster	Represent and solve problems involving addition and subtraction.
M.1.1	Use addition and subtraction within 20 to solve word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions (e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem).
M.1.2	Solve word problems that call for addition of three whole numbers whose sum is less than or equal to 20 (e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem).

Cluster	Understand and apply properties of operations and the relationship between addition and subtraction.
M.1.3	Apply properties of operations as strategies to add and subtract (e.g., If 8 + 3 = 11 is known, then 3 + 8 = 11 is also known: Commutative Property of Addition. To add 2 + 6 + 4, the second two numbers can be added to make a ten, so 2 + 6 + 4 = 2 + 10 = 12: Associative Property of Addition). Instructional Note: Students need not use formal terms for these properties.
M.1.4	Understand subtraction as an unknown-addend problem (e.g., subtract 10 – 8 by finding the number that makes 10 when added to 8).

Cluster	Add and subtract within 20.
M.1.5	Relate counting to addition and subtraction (e.g., by counting on 2 to add 2).
M.1.6	Add and subtract within 20, demonstrating fluency for addition and subtraction within 10 and use strategies such as • counting on; • making ten (e.g., 8 + 6 = 8 + 2 + 4 = 10 + 4 = 14); • decomposing a number leading to a ten (e.g., 13 – 4 = 13 – 3 – 1 = 10 – 1 = 9); • using the relationship between addition and subtraction (e.g., knowing that 8 + 4 = 12, one knows 12 – 8= 4); and • creating equivalent but easier or known sums (e.g., adding 6 +7 by creating the known equivalent 6 + 6 + 1 = 12 + 1 = 13).

Cluster	Work with addition and subtraction equations.
M.1.7	Understand the meaning of the equal sign, and determine if equations involving addition and subtraction are true or false (e.g., Which of the following equations are true and which are false? 6 = 6, 7 = 8 – 1, 5 + 2 = 2 + 5, 4 + 1 = 5 + 2).
M.1.8	Determine the unknown whole number in an addition or subtraction equation relating three whole numbers (e.g., Determine the unknown number that makes the equation true in each of the equations. 8 + ? = 11, 5 = ? – 3, 6 + 6 = ?).

Number and Operations in Base Ten

Cluster	Extend the counting sequence.
M.1.9	Count to 120, starting at any number less than 120. In this range, read and write numerals and represent a number of objects with a written numeral.

Cluster	Understand place value.
M.1.10	Understand the two digits of a two-digit number represent amounts of tens and ones. Understand the following as special cases: a. 10 can be thought of as a bundle of ten ones — called a “ten.” (e.g., A group of ten pennies is equivalent to a dime.) b. The numbers from 11 to 19 are composed of a ten and one, two, three, four, five, six, seven, eight or nine ones. c. The numbers 10, 20, 30, 40, 50, 60, 70, 80, 90 refer to one, two, three, four, five, six, seven, eight or nine tens (and 0 ones).
M.1.11	Compare two two-digit numbers based on meanings of the tens and ones digits, recording the results of comparisons with the symbols >, =, and <.

Cluster	Use place value understanding and properties of operations to add and subtract.
M.1.12	Add within 100, including • adding a two-digit number and a one-digit number and adding a two-digit number and a multiple of 10, • using concrete models or drawings and strategies based on place value, properties of operations and/or the relationship between addition and subtraction. Relate the strategy to a written method and explain the reasoning used. Understand that in adding two-digit numbers, one adds tens and tens, ones and ones, and sometimes it is necessary to compose a ten.
M.1.13	Given a two-digit number, mentally find 10 more or 10 less than the number, without having to count and explain the reasoning used.
M.1.14	Subtract multiples of 10 in the range 10-90 from multiples of 10 in the range 10-90 (positive or zero differences) using concrete models or drawings and strategies based on place value, properties of operations and/or the relationship between addition and subtraction. Relate the strategy to a written method and explain the reasoning used.

Measurement and Data

Cluster	Measure lengths indirectly and by iterating length units.
M.1.15	Order three objects by length and compare the lengths of two objects indirectly by using

	a third object.
M.1.16	Express the length of an object as a whole number of length units, by laying multiple copies of a shorter object (the length unit) end to end; understand that the length measurement of an object is the number of same-size length units that span it with no gaps or overlaps. Instructional Note: Limit to contexts where the object being measured is spanned by a whole number of length units with no gaps or overlaps.

Cluster	**Tell and write time.**
M.1.17	Tell and write time in hours and half-hours using analog and digital clocks.

Cluster	**Represent and interpret data.**
M.1.18	Organize, represent, interpret data with up to three categories; ask and answer questions about the total number of data points, how many in each category and how many more or less are in one category than in another.

Geometry

Cluster	**Reason with shapes and their attributes.**
M.1.19	Distinguish between defining attributes (e.g., triangles are closed and three-sided) versus non-defining attributes (e.g., color, orientation, and/or overall size); build and draw shapes to possess defining attributes.
M.1.20	Compose two-dimensional shapes (rectangles, squares, trapezoids, triangles, half-circles, and quarter-circles) or three-dimensional shapes (cubes, right rectangular prisms, right circular cones, and right circular cylinders) to create a composite shape and compose new shapes from the composite shape. Instructional Note: Students do not need to learn formal names such as, “right rectangular prism.”
M.1.21	Partition circles and rectangles into two and four equal shares, describe the shares using the words halves, fourths and quarters and use the phrases half of, fourth of and quarter of. Describe the whole as two of, or four of the shares and understand for these examples that decomposing into more equal shares creates smaller shares.

Mathematics – Grade 2

All West Virginia teachers are responsible for classroom instruction that integrates content standards and mathematical habits of mind. Students in the second grade will focus on four critical areas: (1) extending understanding of base-ten notation; (2) building fluency with addition and subtraction; (3) using standard units of measure; and (4) describing and analyzing shapes. Mathematical habits of mind, which should be integrated in these content areas, include: making sense of problems and persevering in solving them, reasoning abstractly and quantitatively; constructing viable arguments and critiquing the reasoning of others; modeling with mathematics; using appropriate tools strategically; attending to precision, looking for and making use of structure; and looking for and expressing regularity in repeated reasoning. Continuing the skill progressions from first grade, the following chart represents the mathematical understandings that will be developed in second grade:

Operations and Algebraic Thinking	Number and Operations in Base Ten
• Solve challenging addition and subtraction word problems with one or two steps (e.g., a "one-step" problem would be: "Lucy has 23 fewer apples than Julie. Julie has 47 apples. How many apples does Lucy have?"). • Fluently add with a sum of 20 or less (e.g., 11 + 8); fluently subtract from a number 20 or less (e.g., 16 – 9); and know all sums of one-digit numbers from memory by the end of the year. • Work with equal groups of objects to gain foundations for multiplication.	• Understand what the digits mean in three-digit numbers (place value). • Use an understanding of place value to add and subtract three-digit numbers (e.g., 811 – 367); add and subtract two-digit numbers fluently (e.g., 77 – 28).
Measurement and Data	**Geometry**
• Solve addition and subtraction word problems involving length (e.g., "The pen is 2 cm longer than the pencil. If the pencil is 7 cm long, how long is the pen?"). • Tell time. • Count money.	• Build, draw, and analyze 2-D and 3-D shapes to develop foundations for area, volume, and geometry in later grades. • Divide shapes into equal shares to build the foundations for fractions in later grades.

Numbering of Standards

The following Mathematics Standards will be numbered continuously. The following ranges relate to the clusters found within Mathematics:

Operations and Algebraic Thinking	
Represent and solve problems involving addition and subtraction.	Standard 1
Add and subtract within 20.	Standard 2
Work with equal groups of objects to gain foundations for multiplication.	Standards 3-4
Number and Operations in Base Ten	
Understand place value.	Standard 5-8

Use place value understanding and properties of operations to add and subtract.	Standards 9-13
Measurement and Data	
Measure and estimate lengths in standard units.	Standards 14-17
Relate addition and subtraction to length.	Standards 18-19
Work with time and money.	Standards 20-21
Represent and interpret data.	Standards 22-23
Geometry	
Reason with shapes and their attributes.	Standards 24-26

Operations and Algebraic Thinking

Cluster	Represent and solve problems involving addition and subtraction.
M.2.1	Use addition and subtraction within 100 to solve one- and two-step word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions (e.g. by using drawings and equations with a symbol for the unknown number to represent the problem).

Cluster	Add and subtract within 20.
M.2.2	Fluently add and subtract within 20 using mental strategies and by end of Grade 2, know from memory all sums of two one-digit numbers.

Cluster	Work with equal groups of objects to gain foundations for multiplication.
M.2.3	Determine whether a group of objects (up to 20) has an odd or even number of members, e.g. by pairing objects or counting them by 2s; write an equation to express an even number as a sum of two equal addends.
M.2.4	Use addition to find the total number of objects arranged in rectangular arrays with up to 5 rows and up to 5 columns; write an equation to express the total as a sum of equal addends.

Number and Operations in Base Ten

Cluster	Understand place value.
M.2.5	Understand that the three digits of a three-digit number represent amounts of hundreds, tens and ones (e.g., 706 equals 7 hundreds, 0 tens and 6 ones). Understand the following as special cases: a. 100 can be thought of as a bundle of ten tens – called a “hundred.” b. Numbers 100, 200, 300, 400, 500, 600, 700, 800, 900 refer to one, two, three, four, five, six, seven, eight or nine hundreds, and 0 tens and 0 ones.
M.2.6	Count within 1000 and skip-count by 5s, 10s and 100s.
M.2.7	Read and write numbers to 1000 using base-ten numerals, number names and expanded form.
M.2.8	Compare two three-digit numbers based on meanings of the hundreds, tens and ones digits, using >, = and < symbols to record the results of comparisons.

Cluster	Use place value understanding and properties of operations to add and subtract.
M.2.9	Fluently add and subtract within 100 using strategies based on place value, properties of

	operations and/or the relationship between addition and subtraction.
M.2.10	Add up to four two-digit numbers using strategies based on place value and properties of operations.
M.2.11	Add and subtract within 1000, using concrete models or drawings and strategies based on place value, properties of operations and/or the relationship between addition and subtraction; relate the strategy to a written method. Understand that in adding or subtracting three-digit numbers, one adds or subtracts hundreds and hundreds, tens and tens, ones and ones and sometimes it is necessary to compose or decompose tens or hundreds.
M.2.12	Mentally add 10 or 100 to a given number 100-900 and mentally subtract 10 or 100 from a given number 100-900.
M.2.13	Explain why addition and subtraction strategies work, using place value and the properties of operations. Instructional Note: Explanations may be supported by drawing or objects.

Measurement and Data

Cluster	**Measure and estimate lengths in standard units.**
M.2.14	Measure the length of an object by selecting and using appropriate tools such as rulers, yardsticks, meter sticks, and measuring tapes.
M.2.15	Measure the length of an object twice, using length units of different lengths for the two measurements, describe how the two measurements relate to the size of the unit chosen.
M.2.16	Estimate lengths using units of inches, feet, centimeters, and meters.
M.2.17	Measure to determine how much longer one object is than another, expressing the length difference in terms of a standard length unit.

Cluster	**Relate addition and subtraction to length.**
M.2.18	Use addition and subtraction within 100 to solve word problems involving lengths that are given in the same units (e.g., by using drawings, such as drawings of rulers), and equations with a symbol for the unknown number to represent the problem.
M.2.19	Represent whole numbers as lengths from 0 on a number line diagram with equally spaced points corresponding to the numbers 0, 1, 2… and represent whole-number sums and differences within 100 on a number line diagram.

Cluster	**Work with time and money.**
M.2.20	Tell and write time from analog and digital clocks to the nearest five minutes, using a.m. and p.m.
M.2.21	Solve word problems involving dollar bills, quarters, dimes, nickels, and pennies, using $ and ¢ symbols appropriately (e.g., If you have 2 dimes and 3 pennies, how many cents do you have?).

Cluster	**Represent and interpret data.**
M.2.22	Generate measurement data by measuring lengths of several objects to the nearest whole unit or by making repeated measurements of the same object. Show the measurements by making a line plot, where the horizontal scale is marked off in whole-number units.

M.2.23	Draw a picture graph and a bar graph (with single-unit scale) to represent a data set with up to four categories. Solve simple put-together, take-apart, and compare problems using information presented in a bar graph.

Geometry

Cluster	**Reason with shapes and their attributes**
M.2.24	Recognize and draw shapes having specified attributes, such as a given number of angles or a given number of equal faces (sizes are compared directly or visually, not compared by measuring). Identify triangles, quadrilaterals, pentagons, hexagons, and cubes.
M.2.25	Partition a rectangle into rows and columns of same-size squares and count to find the total number of them.
M.2.26	Partition circles and rectangles into two, three, or four equal shares, describe the shares using the words halves, thirds, half of, a third of, etc., describe the whole as two halves, three thirds, four fourths. Recognize that equal shares of identical wholes need not have the same shape.

Mathematics – Grade 3

All West Virginia teachers are responsible for classroom instruction that integrates content standards and mathematical habits of mind. Students in the third grade will focus on four critical areas: (1) developing understanding of multiplication and division and strategies for multiplication and division within 100; (2) developing understanding of fractions, especially unit fractions (fractions with numerator 1); (3) developing understanding of the structure of rectangular arrays and of area; and (4) describing and analyzing two-dimensional shapes. Mathematical habits of mind, which should be integrated in these content areas, include: making sense of problems and persevering in solving them, reasoning abstractly and quantitatively; constructing viable arguments and critiquing the reasoning of others; modeling with mathematics; using appropriate tools strategically; attending to precision, looking for and making use of structure; and looking for and expressing regularity in repeated reasoning. Continuing the skill progressions from second grade, the following chart represents the mathematical understandings that will be developed in third grade:

<table>
<tr><th>Operations and Algebraic Thinking</th><th>Number and Operations in Base Ten</th></tr>
<tr><td>• Understand and know from memory how to multiply and divide numbers up to 10 x 10 fluently.
• Solve word problems using addition, subtraction, multiplication, and division.
• Begin to multiply numbers with more than one digit (e.g., multiplying 9 × 80).</td><td>• Understand place value and properties of operations to perform multi-digit arithmetic, such as 10 x 2, 50 x 3, and 40 x 7.</td></tr>
<tr><th>Number and Operations- Fractions</th><th>Measurement and Data</th></tr>
<tr><td>• Understand fractions and relate them to the familiar system of whole numbers (e.g., recognizing that 3/1 and 3 are the same number).</td><td rowspan="3">• Measure and estimate weights and liquid volumes, and solve word problems involving these quantities.
• Tell time and write time to the nearest minute.
• Recognize area as a quality of two-dimensional regions.
• Understand that rectangular arrays can be broken into identical rows or into identical columns. By breaking rectangles into rectangular arrays of squares, students connect area to multiplication, and explain how multiplication is used to determine the area of a rectangle.</td></tr>
<tr><th>Geometry</th></tr>
<tr><td>• Reason about shapes (e.g., all squares are rectangles but not all rectangles are squares).
• Find areas of shapes, and relate area to multiplication (e.g., why is the number of square feet for a 9-foot by 7-foot room given by the product 9 × 7?).
• Understand the connection between equal parts of a shape being a unit of the whole.</td></tr>
</table>

<u>Numbering of Standards</u>

The following Mathematics Standards will be numbered continuously. The following ranges relate to the clusters found within Mathematics:

Operations and Algebraic Thinking	
Represent and solve problems involving multiplication and division.	Standards 1-4

Understand properties of multiplication and the relationship between multiplication and division.	Standards 5-6
Multiply and divide within 100.	Standard 7
Solve problems involving the four operations, and identify and explain patterns in arithmetic.	Standards 8-9
Number and Operations in Base Ten	
Use place value and properties of operations to perform multi-digit arithmetic.	Standards 10-12
Number and Operations- Fractions	
Develop an understanding as fractions as numbers.	Standards 13-15
Measurement and Data	
Solve problems involving measurement and estimation of intervals of time, liquid volumes, and masses of objects.	Standards 16-17
Represent and interpret data.	Standards 18-19
Geometric measurement: understand concepts of area and relate area to multiplication and to addition.	Standards 20-22
Geometric measurement: recognize perimeter as an attribute of plane figures and distinguish between linear and area measures.	Standard 23
Geometry	
Reason with shapes and their attributes.	Standards 24-25

Operations and Algebraic Thinking

Cluster	Represent and solve problems involving multiplication and division.
M.3.1	Interpret products of whole numbers, e.g., interpret 5 × 7 as the total number of objects in 5 groups of 7 objects each (e.g., describe context in which a total number of objects can be expressed as 5 × 7).
M.3.2	Interpret whole-number quotients of whole numbers, e.g., interpret 56 ÷ 8 as the number of objects in each share when 56 objects are partitioned equally into 8 shares, or as a number of shares when 56 objects are partitioned into equal shares of 8 objects each (e.g., describe a context in which a number of shares or a number of groups can be expressed as 56 ÷ 8).
M.3.3	Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays and measurement quantities (e.g., by using drawings and equations with a symbol for the unknown number to represent the problem).
M.3.4	Determine the unknown whole number in a multiplication or division equation relating three whole numbers (e.g., determine the unknown number that makes the equation true in each of the equations 8 × ? = 48, 5 = ? ÷ 3, 6 × 6 =?).

Cluster	Understand properties of multiplication and the relationship between multiplication and division.
M.3.5	Apply properties of operations as strategies to multiply and divide (e.g., If 6 × 4 = 24 is known, then 4 × 6 = 24 is also known: Commutative Property of Multiplication. 3 × 5 × 2

	can be found by 3 × 5 = 15, then 15 × 2 = 30, or by 5 × 2 = 10, then 3 × 10 = 30: Associative Property of Multiplication. Knowing that 8 × 5 = 40 and 8 × 2 = 16, one can find 8 × 7 as 8 × (5 + 2) = (8 × 5) + (8 × 2) = 40 + 16 = 56: Distributive Property. Instructional Note: Students need not use formal terms for these properties.
M.3.6	Understand division as an unknown-factor problem (e.g., find 32 ÷ 8 by finding the number that makes 32 when multiplied by 8).

Cluster	**Multiply and divide within 100.**
M.3.7	Learn multiplication tables (facts) with speed and memory in order to fluently multiply and divide within 100, using strategies such as the relationship between multiplication and division (e.g., knowing that 8 × 5 = 40, one knows that 40 ÷ 5 = 8) or properties of operations by the end of Grade 3.

Cluster	**Solve problems involving the four operations, and identify and explain patterns in arithmetic.**
M.3.8	Solve two-step word problems using the four operations, represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding. Instructional Note: This standard is limited to problems posed with whole numbers and having whole number answers; students should know how to perform operations in the conventional order when there are no parentheses to specify a particular order (Order of Operations).
M.3.9	Identify arithmetic patterns (including patterns in the addition table or multiplication table) and explain those using properties of operations (e.g., observe that 4 times a number is always even and explain why 4 times a number can be decomposed into two equal addends).

Number and Operations in Base Ten

Cluster	**Use place value understanding and properties of operations to perform multi-digit arithmetic.**
M.3.10	Use place value understanding to round whole numbers to the nearest 10 or 100.
M.3.11	Fluently add and subtract within 1000 using strategies and algorithms based on place value, properties of operations, and/or the relationship between addition and subtraction.
M.3.12	Multiply one-digit whole numbers by multiples of 10 in the range 10–90 (e.g., 9 × 80, 5 × 60) using strategies based on place value and properties of operations.

Number and Operations- Fractions

Cluster	**Develop understanding of fractions as numbers.**
M.3.13	Understand a fraction 1/b as the quantity formed by 1 part when a whole is partitioned into b equal parts; understand a fraction a/b as the quantity formed by a parts of size 1/b. Instructional Note: Fractions in this standard are limited to denominators of 2, 3, 4, 6, and 8.
M.3.14	Understand a fraction as a number on the number line and represent fractions on a number line diagram.

	a. Represent a fraction 1/b on a number line diagram by defining the interval from 0 to 1 as the whole and partitioning it into b equal parts. Recognize that each part has size 1/b and that the endpoint of the part based at 0 locates the number 1/b on the number line. (e.g., Given that b parts is 4 parts, then 1/b represents 1/4. Students partition the number line into fourths and locate 1/4 on the number line.) b. Represent a fraction a/b on a number line diagram by marking off a lengths 1/b from 0. Recognize that the resulting interval has size a/b and that its endpoint locates the number a/b on the number line. (e.g., Given that a/b represents 3/4 or 6/4, students partition the number line into fourths and represent these fractions accurately on the same number line; students extend the number line to include the number of wholes required for the given fractions.) Instructional Note: Fractions in this standard are limited to denominators of 2, 3, 4, 6, and 8.
M.3.15	Explain equivalence of fractions in special cases and compare fractions by reasoning about their size. a. Understand two fractions as equivalent (equal) if they are the same size or the same point on a number line. b. Recognize and generate simple equivalent fractions (e.g., 1/2 = 2/4, 4/6 = 2/3). Explain why the fractions are equivalent (e.g., by using a visual fraction model). c. Express whole numbers as fractions, and recognize fractions that are equivalent to whole numbers. (e.g., Express 3 in the form 3 = 3/1; recognize that 6/1 = 6; locate 4/4 and 1 at the same point of a number line diagram.) d. Compare two fractions with the same numerator or the same denominator by reasoning about their size. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with the symbols >, = or < and justify the conclusions (e.g., by using a visual fraction model). Instructional Note: Fractions in this standard are limited to denominators of 2, 3, 4, 6, and 8.

Measurement and Data

Cluster	**Solve problems involving measurement and estimation of intervals of time, liquid volumes, and masses of objects.**
M.3.16	Tell and write time to the nearest minute, measure time intervals in minutes. Solve word problems involving addition and subtraction of time intervals in minutes (e.g., by representing the problem on a number line diagram).
M.3.17	Measure and estimate liquid volumes and masses of objects using standard units of grams (g), kilograms (kg) and liters (l). Add, subtract, multiply or divide to solve one-step word problems involving masses or volumes that are given in the same units (e.g., by using drawings, such as a beaker with a measurement scale) to represent the problem. Instructional Note: Exclude compound units such as cm^3 and finding the geometric volume of a container.

Cluster	**Represent and interpret data.**
M.3.18	Draw a scaled picture graph and a scaled bar graph to represent a data set with several

	categories. Solve one- and two-step "how many more" and "how many less" problems using information presented in scaled bar graphs (e.g., draw a bar graph in which each square in the bar graph might represent 5 pets).
M.3.19	Generate measurement data by measuring lengths using rulers marked with halves and fourths of an inch. Show the data by making a line plot, where the horizontal scale is marked off in appropriate units—whole numbers, halves or quarters.

Cluster	Geometric measurement: understand concepts of area and relate area to multiplication and to addition.
M.3.20	Recognize area as an attribute of plane figures and understand concepts of area measurement. a. A square with side length 1 unit, called "a unit square," is said to have "one square unit" of area and can be used to measure area. b. A plane figure which can be covered without gaps or overlaps by b unit squares is said to have an area of b square units.
M.3.21	Measure areas by counting unit squares (square cm, square m, square in, square ft. and improvised units).
M.3.22	Relate area to the operations of multiplication and addition. a. Find the area of a rectangle with whole-number side lengths by tiling it, and show that the area is the same as would be found by multiplying the side lengths. b. Multiply side lengths to find areas of rectangles with whole number side lengths in the context of solving real world and mathematical problems, and represent whole-number products as rectangular areas in mathematical reasoning. c. Use tiling to show in a concrete case that the area of a rectangle with whole-number side lengths a and b + c is the sum of a × b and a × c. Use area models to represent the distributive property in mathematical reasoning. d. Recognize area as additive and find areas of rectilinear figures by decomposing them into non-overlapping rectangles and adding the areas of the non-overlapping parts, applying this technique to solve real world problems.

Cluster	Geometric measurement: recognize perimeter as an attribute of plane figures and distinguish between linear and area measures.
M.3.23	Solve real world and mathematical problems involving perimeters of polygons, including finding the perimeter given the side lengths, finding an unknown side length and exhibiting rectangles with the same perimeter and different areas or with the same area and different perimeters.

Geometry

Cluster	Reason with shapes and their attributes.
M.3.24	Understand that shapes in different categories (e.g., rhombuses, rectangles, and others) may share attributes (e.g., having four sides), that the shared attributes can define a larger category (e.g. quadrilaterals). Recognize rhombuses, rectangles, and squares as examples of quadrilaterals, and draw examples of quadrilaterals that do not belong to any of these subcategories.
M.3.25	Partition shapes into parts with equal areas. Express the area of each part as a unit

	fraction of the whole. For example, partition a shape into 4 parts with equal area, and describe the area of each part as ¼ or the area of the shape.

Mathematics – Grade 4

All West Virginia teachers are responsible for classroom instruction that integrates content standards and mathematical habits of mind. Students in the fourth grade will focus on three critical areas: (1) developing understanding and fluency with multi-digit multiplication, and developing understanding of dividing to find quotients involving multi-digit dividends; (2) developing an understanding of fraction equivalence, addition and subtraction of fractions with like denominators, and multiplication of fractions by whole numbers; (3) understanding that geometric figures can be analyzed and classified based on their properties, such as having parallel sides, perpendicular sides, particular angle measures, and symmetry. Mathematical habits of mind, which should be integrated in these content areas, include: making sense of problems and persevering in solving them, reasoning abstractly and quantitatively; constructing viable arguments and critiquing the reasoning of others; modeling with mathematics; using appropriate tools strategically; attending to precision, looking for and making use of structure; and looking for and expressing regularity in repeated reasoning. Continuing the skill progressions from third grade, the following chart represents the mathematical understandings that will be developed in fourth grade:

Operations and Algebraic Thinking	**Number and Operations in Base Ten**
• Use whole-number arithmetic to solve word problems, including problems with remainders and problems with measurements. • Add and subtract whole numbers quickly and accurately (numbers up to 1 million). • Multiply and divide multi-digit numbers in simple cases (e.g., multiplying 1,638 × 7 or 24 × 17, and dividing 6,966 by 6). • Gain familiarity with factors and multiples. • Generate and analyze patterns.	• Generalize place value understanding for multi-digit whole numbers. • Use place value understanding and properties of operations to perform multi-digit arithmetic.
Number and Operations- Fractions	**Measurement and Data**
• Use equivalent fractions to understand and order fractions (e.g., recognize that 1⁄4 is less than 3⁄8 because 2⁄8 is less than 3⁄8). • Add, subtract, and multiply fractions in simple cases (such as 2 3⁄4 − 1 1⁄4 or 3 × 5⁄8), and solve related word problems. • Understand and compare simple decimals in terms of fractions (e.g., rewriting 0.62 as 62⁄100).	• Solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit. • Represent and interpret data. • Geometric measurement: understand concepts of angle and measure angles.
Geometry	
• Draw and identify lines and angles, and classify shapes by properties of their lines and angles. • Measure angles and find unknown angles in a diagram.	

Numbering of Standards

The following Mathematics Standards will be numbered continuously. The following ranges relate to the clusters found within Mathematics:

Operations and Algebraic Thinking	
Use the four operations with whole numbers to solve problems.	Standards 1-3
Gain familiarity with factors and multiples.	Standard 4
Generate and analyze patterns.	Standard 5
Number and Operations in Base Ten	
Generalize place value understanding for multi-digit whole numbers.	Standards 6-8
Use place value understanding and properties of operations to perform multi-digit arithmetic.	Standards 9-11
Number and Operations- Fractions	
Extend understanding of fraction equivalence and ordering.	Standards 12-13
Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers.	Standards 14-15
Understand decimal notation for fractions, and compare decimal fractions.	Standards 16-18
Measurement and Data	
Solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit.	Standards 19-21
Represent and interpret data.	Standards 22
Geometric measurement: understand concepts of angle and measure angles.	Standards 23-25
Geometry	
Draw and identify lines angles and classify shapes by properties of their lines and angles.	Standards 26-28

Operations and Algebraic Thinking

Cluster	Use the four operations with whole numbers to solve problems.
M.4.1	Interpret a multiplication equation as a comparison (e.g., interpret 35 = 5 × 7 as a statement that 35 is 5 times as many as 7 and 7 times as many as 5). Represent verbal statements of multiplicative comparisons as multiplication equations.
M.4.2	Multiply or divide to solve word problems involving multiplicative comparison (e.g., by using drawings and equations with a symbol for the unknown number to represent the problem) and distinguish multiplicative comparison from additive comparison.
M.4.3	Solve multi-step word problems posed with whole numbers and having whole-number answers using the four operations, including problems in which remainders must be interpreted. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and

	estimation strategies including rounding.

Cluster	Gain familiarity with factors and multiples.
M.4.4	Find all factor pairs for a whole number in the range 1–100, recognize that a whole number is a multiple of each of its factors. Determine whether a given whole number in the range 1–100 is a multiple of a given one-digit number. Determine whether a given whole number in the range 1–100 is prime or composite.

Cluster	Generate and analyze patterns.
M.4.5	Generate a number or shape pattern that follows a given rule. Identify apparent features of the pattern that were not explicit in the rule itself. (e.g., Given the rule "Add 3" and the starting number 1, generate terms in the resulting sequence and observe that the terms appear to alternate between odd and even numbers. Explain informally why the numbers will continue to alternate in this way.)

Number and Operations in Base Ten

Cluster	Generalize place value understanding for multi-digit whole numbers.
M.4.6	Recognize that in a multi-digit whole number, a digit in one place represents ten times what it represents in the place to its right (e.g., recognize that 700 ÷ 70 = 10 by applying concepts of place value and division).
M.4.7	Read and write multi-digit whole numbers using base-ten numerals, number names, and expanded form. Compare two multi-digit numbers based on meanings of the digits in each place, using >, = and < symbols to record the results of comparisons.
M.4.8	Use place value understanding to round multi-digit whole numbers to any place.

Cluster	Use place value understanding and properties of operations to perform multi-digit arithmetic.
M.4.9	Fluently add and subtract multi-digit whole numbers using the standard algorithm.
M.4.10	Multiply a whole number of up to four digits by a one-digit whole number, multiply two two-digit numbers, using strategies based on place value and the properties of operations and illustrate and explain the calculation by using equations, rectangular arrays and/or area models.
M.4.11	Find whole-number quotients and remainders with up to four-digit dividends and one-digit divisors, using strategies based on place value, the properties of operations and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays and/or area models.

Number and Operations- Fractions

Cluster	Extend understanding of fraction equivalence and ordering.
M.4.12	Explain why a fraction a/b is equivalent to a fraction (n × a)/(n × b) by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions.
M.4.13	Compare two fractions with different numerators and different denominators (e.g., by creating common denominators or numerators, or by comparing to a benchmark

	fraction such as ½). Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols >, = or <, and justify the conclusions by using a visual fraction model.

Cluster	**Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers.**
M.4.14	Understand the fraction a/b, with a > 1, as the sum of a of the fractions 1/b. a. Understand addition and subtraction of fractions as joining and separating parts referring to the same whole. b. Decompose a fraction into a sum of fractions with the same denominator in more than one way, recording each decomposition by an equation and justify decompositions by using a visual fraction model (e.g., 3/8 = 1/8 + 1/8 + 1/8; 3/8 = 1/8 + 2/8; 2 1/8 = 1 + 1 + 1/8 = 8/8 + 8/8 + 1/8). c. Add and subtract mixed numbers with like denominators by replacing each mixed number with an equivalent fraction and/or by using properties of operations and the relationship between addition and subtraction. d. Solve word problems involving addition and subtraction of fractions referring to the same whole and having like denominators by using visual fraction models and equations to represent the problem.
M.4.15	Apply and extend previous understandings of multiplication to multiply a fraction by a whole number. a. Understand a fraction a/b as a multiple of 1/b, (e.g., use a visual fraction model to represent 5/4 as the product 5 × (1/4), recording the conclusion by the equation 5/4 = 5 × (1/4)). b. Understand a multiple of a/b as a multiple of 1/b, and use this understanding to multiply a fraction by a whole number (e.g., use a visual fraction model to express 3 × (2/5) as 6 × (1/5), recognizing this product as 6/5. In general, n × (a/b) = (n × a)/b). c. Solve word problems involving multiplication of a fraction by a whole number by using visual fraction models and equations to represent the problem (e.g., If each person at a party will eat 3/8 of a pound of roast beef, and there will be 5 people at the party, how many pounds of roast beef will be needed? Between what two whole numbers does your answer lie?).

Cluster	**Understand decimal notation for fractions, and compare decimal fractions.**
M.4.16	Express a fraction with denominator 10 as an equivalent fraction with denominator 100, and use this technique to add two fractions with respective denominators 10 and 100 (e.g., express 3/10 as 30/100, and add 3/10 + 4/100 = 34/100). Instructional Note: Students who can generate equivalent fractions can develop strategies for adding fractions with unlike denominators in general. But addition and subtraction with unlike denominators in general is not a requirement at this grade.
M.4.17	Use decimal notation for fractions with denominators 10 or 100 (e.g., rewrite 0.62 as 62/100; describe a length as 0.62 meters; locate 0.62 on a number line diagram).
M.4.18	Compare two decimals to hundredths by reasoning about their size. Recognize that comparisons are valid only when the two decimals refer to the same whole. Record the results of comparisons with the symbols >, = or <, and justify the conclusions by using a visual model.

Measurement and Data

Cluster	**Solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit.**
M.4.19	Know relative sizes of measurement units within a system of units, including the metric system (km, m, cm; kg, g; l, ml), the standard system (lb, oz), and time (hr, min, sec.). Within a single system of measurement, express measurements in a larger unit in terms of a smaller unit. Record measurement equivalents in a two-column table. (e.g., Know that 1 ft is 12 times as long as 1 in. Express the length of a 4 ft snake as 48 in. Generate a conversion table for feet and inches listing the number pairs (1, 12), (2, 24), (3, 36), ...)
M.4.20	Use the four operations to solve word problems involving distances, intervals of time, liquid volumes, masses of objects, and money, including problems involving simple fractions or decimals and problems that require expressing measurements given in a larger unit in terms of a smaller unit. Represent measurement quantities using diagrams such as number line diagrams that feature a measurement scale.
M.4.21	Apply the area and perimeter formulas for rectangles in real world and mathematical problems by viewing the area formula as a multiplication equation with an unknown factor. (e.g., find the width of a rectangular room given the area of the flooring and the length.)

Cluster	**Represent and interpret data.**
M.4.22	Make a line plot to display a data set of measurements in fractions of a unit (1/2, 1/4, 1/8). Solve problems involving addition and subtraction of fractions by using information presented in line plots (e.g., from a line plot find and interpret the difference in length between the longest and shortest specimens in an insect collection).

Cluster	**Geometric measurement: understand concepts of angle and measure angles.**
M.4.23	Recognize angles as geometric shapes that are formed wherever two rays share a common endpoint, and understand concepts of angle measurement: a. An angle is measured with reference to a circle with its center at the common endpoint of the rays, by considering the fraction of the circular arc between the points where the two rays intersect the circle. An angle that turns through 1/360 of a circle is called a "one-degree angle," and can be used to measure angles. b. An angle that turns through b one-degree angles is said to have an angle measure of b degrees.
M.4.24	Measure angles in whole-number degrees using a protractor and sketch angles of specified measure.
M.4.25	Recognize angle measure as additive. When an angle is decomposed into non-overlapping parts, the angle measure of the whole is the sum of the angle measures of the parts. Solve addition and subtraction problems to find unknown angles on a diagram in real world and mathematical problems (e.g., by using an equation with a symbol for the unknown angle measure).

Geometry

Cluster	**Draw and identify lines and angles and classify shapes by properties of their lines and angles.**
M.4.26	Draw points, lines, line segments, rays, angles (right, acute, obtuse) and perpendicular and parallel lines. Identify these in two-dimensional figures.
M.4.27	Classify two-dimensional figures based on the presence or absence of parallel or perpendicular lines or the presence or absence of angles of a specified size. Recognize right triangles as a category, and identify right triangles.
M.4.28	Recognize a line of symmetry for a two-dimensional figure as a line across the figure such that the figure can be folded along the line into matching parts. Identify line-symmetric figures and draw lines of symmetry.

Mathematics – Grade 5

All West Virginia teachers are responsible for classroom instruction that integrates content standards and mathematical habits of mind. Students in the fifth grade will focus on three critical areas: (1) developing fluency with addition and subtraction of fractions, and developing understanding of the multiplication of fractions and of division of fractions in limited cases (unit fractions divided by whole numbers and whole numbers divided by unit fractions); (2) extending division to 2-digit divisors, integrating decimal fractions into the place value system and developing understanding of operations with decimals to hundredths, and developing fluency with whole number and decimal operations; (3) developing an understanding of volume. Mathematical habits of mind, which should be integrated in these content areas, include: making sense of problems and persevering in solving them, reasoning abstractly and quantitatively; constructing viable arguments and critiquing the reasoning of others; modeling with mathematics; using appropriate tools strategically; attending to precision, looking for and making use of structure; and looking for and expressing regularity in repeated reasoning. Students in fifth grade will continue developing mathematical proficiency in a developmentally-appropriate progressions of standards. Continuing the skill progressions from fourth grade, the following chart represents the mathematical understandings that will be developed in fifth grade:

Operations and Algebraic Thinking	**Number and Operations in Base Ten**
• Write and interpret numerical expressions. • Analyze mathematical patterns and relationships.	• Understand the place value system. • Generalize the place-value system to include decimals, and calculate with decimals to the hundredths place (two places after the decimal). • Multiply whole numbers quickly and accurately, for example 1,638 × 753, and divide whole numbers in simple cases, such as dividing 6,971 by 63.
Number and Operations- Fractions	**Measurement and Data**
• Add and subtract fractions with like and unlike denominators (e.g., 21⁄4 – 11⁄3), and solve word problems of this kind. • Multiply fractions; divide fractions in simple cases; and solve related word problems (e.g., find the area of a rectangle with fractional side lengths; determine how many 1⁄3-cup servings are in 2 cups of raisins; determine the size of a share if 9 people share a 50-pound sack of rice equally or if 3 people share 1⁄2 pound of chocolate equally).	• Convert like measurement units within a given measurement system. • Make a line plot to display a data set with fractional units of measure and interpret the data to solve problems. • Geometric measurement: Understand the concept of volume, and solve word problems that involve volume.
Geometry	
• Graph points on the coordinate plane to solve real-world and mathematical problems. • Classify two-dimensional figures into categories based on their properties.	

<u>Numbering of Standards</u>

The following Mathematics Standards will be numbered continuously. The following ranges relate to the clusters found within Mathematics:

Operations and Algebraic Thinking	
Write and interpret numerical expressions.	Standards 1-2
Analyze patterns and relationships.	Standard 3
Number and Operations in Base Ten	
Understand the place value system.	Standard 4-7
Perform operations with multi-digit whole numbers and with decimals to hundredths.	Standards 8-10
Number and Operations- Fractions	
Use equivalent fractions as a strategy to add and subtract fractions.	Standards 11-12
Apply and extend previous understandings of multiplication and division to multiply and divide fractions.	Standards 13-17
Measurement and Data	
Convert like measurement units within a given measurement system.	Standard 18
Represent and interpret data.	Standard 19
Geometric measurement: understand concepts of volume and relate volume to multiplication and to addition.	Standards 20-22
Geometry	
Graph points on the coordinate plane to solve real-world and mathematical problems.	Standards 23-24
Classify two-dimensional figures into categories based on their properties.	Standards 25-26

Operations and Algebraic Thinking

Cluster	**Write and Interpret numerical expressions.**
M.5.1	Use parentheses, brackets or braces in numerical expressions and evaluate expressions with these symbols.
M.5.2	Write simple expressions that record calculations with numbers and interpret numerical expressions without evaluating them. (e.g., Express the calculation “add 8 and 7, then multiply by 2” as 2 × (8 + 7). Recognize that 3 × (18932 + 921) is three times as large as 18932 + 921, without having to calculate the indicated sum or product.)

Cluster	**Analyze patterns and relationships**
M.5.3	Generate two numerical patterns using two given rules. Identify apparent relationships between corresponding terms. Form ordered pairs consisting of corresponding terms from the two patterns, and graph the ordered pairs on a coordinate plane. (e.g., Given the rule “Add 3” and the starting number 0 and given the rule “Add 6” and the starting number 0, generate terms in the resulting sequences and observe that the terms in one sequence are twice the corresponding terms in the other sequence. Explain informally

	why this is so.)

Number and Operations in Base Ten

Cluster	**Understand the place value system**
M.5.4	Recognize that in a multi-digit number, a digit in one place represents 10 times as much as it represents in the place to its right and 1/10 of what it represents in the place to its left.
M.5.5	Explain patterns in the number of zeros of the product when multiplying a number by powers of 10, explain patterns in the placement of the decimal point when a decimal is multiplied or divided by a power of 10. Use whole-number exponents to denote powers of 10.
M.5.6	Read, write, and compare decimals to thousandths. a. Read and write decimals to thousandths using base-ten numerals, number names and expanded form (e.g., 347.392 = 3 × 100 + 4 × 10 + 7 × 1 + 3 × (1/10) + 9 × (1/100) + 2 × (1/1000)). b. Compare two decimals to thousandths based on meanings of the digits in each place, using >, = and < symbols to record the results of comparisons.
M.5.7	Use place value understanding to round decimals to any place.

Cluster	**Perform operations with multi-digit whole numbers and with decimals to hundredths.**
M.5.8	Fluently multiply multi-digit whole numbers using the standard algorithm.
M.5.9	Find whole-number quotients of whole numbers with up to four-digit dividends and two-digit divisors, using strategies based on place value, the properties of operations and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.
M.5.10	Add, subtract, multiply and divide decimals to hundredths, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between related operations, relate the strategy to a written method and explain the reasoning used.

Number and Operations - Fractions

Cluster	**Use equivalent fractions as a strategy to add and subtract fractions.**
M.5.11	Add and subtract fractions with unlike denominators, including mixed numbers, by replacing given fractions with equivalent fractions in such a way as to produce an equivalent sum or difference of fractions with like denominators (e.g., 2/3 + 5/4 = 8/12 + 15/12 = 23/12). Instructional Note: In general, a/b + c/d = (ad + bc)/bd.
M.5.12	Solve word problems involving addition and subtraction of fractions referring to the same whole, including cases of unlike denominators by using visual fraction models or equations to represent the problem. Use benchmark fractions and number sense of fractions to estimate mentally and assess the reasonableness of answers (e.g., recognize an incorrect result 2/5 + 1/2 = 3/7, by observing that 3/7 < 1/2).

Cluster	**Apply and extend previous understandings of multiplication and division to multiply and divide fractions.**
M.5.13	Interpret a fraction as division of the numerator by the denominator (a/b = a ÷ b). Solve

	word problems involving division of whole numbers leading to answers in the form of fractions or mixed numbers by using visual fraction models or equations to represent the problem. (e.g., Interpret 3/4 as the result of dividing 3 by 4, noting that 3/4 multiplied by 4 equals 3 and that when 3 wholes are shared equally among 4 people each person has a share of size 3/4. If 9 people want to share a 50-pound sack of rice equally by weight, how many pounds of rice should each person get? Between what two whole numbers does your answer lie?)
M.5.14	Apply and extend previous understandings of multiplication to multiply a fraction or whole number by a fraction. a. Interpret the product (a/b) × q as a parts of a partition of q into b equal parts; equivalently, as the result of a sequence of operations a × q ÷ b. (e.g., Use a visual fraction model to show (2/3) × 4 = 8/3 and create a story context for this equation. Do the same with (2/3) × (4/5) = 8/15.) Instructional Note: In general, (a/b) × (c/d) = ac/bd. b. Find the area of a rectangle with fractional side lengths by tiling it with unit squares of the appropriate unit fraction side lengths and show that the area is the same as would be found by multiplying the side lengths. Multiply fractional side lengths to find areas of rectangles and represent fraction products as rectangular areas.
M.5.15	Interpret multiplication as scaling (resizing), by: a. Comparing the size of a product to the size of one factor on the basis of the size of the other factor, without performing the indicated multiplication. b. Explaining why multiplying a given number by a fraction greater than 1 results in a product greater than the given number (recognizing multiplication by whole numbers greater than 1 as a familiar case); explaining why multiplying a given number by a fraction less than 1 results in a product smaller than the given number; and relating the principle of fraction equivalence a/b = (n×a)/(n×b) to the effect of multiplying a/b by 1.
M.5.16	Solve real-world problems involving multiplication of fractions and mixed numbers by using visual fraction models or equations to represent the problem.
M.5.17	Apply and extend previous understandings of division to divide unit fractions by whole numbers and whole numbers by unit fractions. Instructional Note: Students able to multiply fractions in general can develop strategies to divide fractions in general, by reasoning about the relationship between multiplication and division, but division of a fraction by a fraction is not a requirement at this grade. a. Interpret division of a unit fraction by a non-zero whole number and compute such quotients. (e.g., Create a story context for (1/3) ÷ 4 and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that (1/3) ÷ 4 = 1/12 because (1/12) × 4 = 1/3.) b. Interpret division of a whole number by a unit fraction and compute such quotients. (e.g., Create a story context for 4 ÷ (1/5) and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that 4 ÷ (1/5) = 20 because 20 × (1/5) = 4.) c. Solve real-world problems involving division of unit fractions by non-zero whole numbers and division of whole numbers by unit fractions by using visual fraction models and equations to represent the problem. (e.g., How much chocolate will each person get if 3 people share 1/2 lb. of chocolate equally? How many1/3-cup servings are in 2 cups of raisins?)

Measurement and Data

Cluster	Convert like measurement units within a given measurement system.
M.5.18	Convert among different-sized standard measurement units within a given measurement system (e.g., convert 5 cm to 0.05 m) and use these conversions in solving multi-step, real-world problems.

Cluster	Represent and interpret data.
M.5.19	Make a line plot to display a data set of measurements in fractions of a unit (1/2, 1/4, 1/8). Use operations on fractions for this grade to solve problems involving information presented in line plots. (e.g., Given different measurements of liquid in identical beakers, find the amount of liquid each beaker would contain if the total amount in all the beakers were redistributed equally).

Cluster	Geometric measurement: understand concepts of volume and relate volume to multiplication and to addition.
M.5.20	Recognize volume as an attribute of solid figures and understand concepts of volume measurement. a. A cube with side length 1 unit, called a “unit cube,” is said to have “one cubic unit” of volume and can be used to measure volume. b. A solid figure which can be packed without gaps or overlaps using b unit cubes is said to have a volume of b cubic units.
M.5.21	Measure volumes by counting unit cubes, using cubic cm, cubic in, cubic ft, and improvised units.
M.5.22	Relate volume to the operations of multiplication and addition and solve real-world and mathematical problems involving volume. a. Find the volume of a right rectangular prism with whole-number side lengths by packing it with unit cubes and show that the volume is the same as would be found by multiplying the edge lengths, equivalently by multiplying the height by the area of the base. Represent threefold whole-number products as volumes (e.g., to represent the associative property of multiplication). b. Apply the formulas $V = l \times w \times h$ and $V = b \times h$ for rectangular prisms to find volumes of right rectangular prisms with whole number edge lengths in the context of solving real-world and mathematical problems. c. Recognize volume as additive and find volumes of solid figures composed of two non-overlapping right rectangular prisms by adding the volumes of the non-overlapping parts, applying this technique to solve real-world problems.

Geometry

Cluster	Graph points on the coordinate plane to solve real-world and mathematical problems.
M.5.23	Use a pair of perpendicular number lines, called axes, to define a coordinate system, with the intersection of the lines, the origin, arranged to coincide with the 0 on each line and a given point in the plane located by using an ordered pair of numbers, called its coordinates. Understand that the first number indicates how far to travel from the origin in the direction of one axis and the second number indicates how far to travel in

	the direction of the second axis, with the convention that the names of the two axes and the coordinates correspond (e.g., x-axis and x-coordinate, y-axis and y-coordinate).
M.5.24	Represent real-world mathematical problems by graphing points in the first quadrant of the coordinate plane and interpret coordinate values of points in the context of the situation.

Cluster	**Classify two-dimensional figures into categories based on their properties.**
M.5.25	Understand that attributes belonging to a category of two dimensional figures also belong to all subcategories of that category (e.g., all rectangles have four right angles and squares are rectangles, so all squares have four right angles).
M.5.26	Classify two-dimensional figures in a hierarchy based on properties.

Mathematics – Grade 6

All West Virginia teachers are responsible for classroom instruction that integrates content standards and mathematical habits of mind. Students in the sixth grade will focus on four critical areas: (1) connecting ratio and rate to whole number multiplication and division and using concepts of ratio and rate to solve problems; (2) completing understanding of division of fractions and extending the notion of number to the system of rational numbers, which includes negative numbers; (3) writing, interpreting and using expressions and equations; and (4) developing understanding of statistical thinking. Mathematical habits of mind, which should be integrated in these content areas, include: making sense of problems and persevering in solving them, reasoning abstractly and quantitatively; constructing viable arguments and critiquing the reasoning of others; modeling with mathematics; using appropriate tools strategically; attending to precision, looking for and making use of structure; and looking for and expressing regularity in repeated reasoning. Students in sixth grade will continue developing mathematical proficiency in a developmentally-appropriate progressions of standards. Continuing the skill progressions from fifth grade, the following chart represents the mathematical understandings that will be developed in sixth grade:

Ratios and Proportional Reasoning	**The Number System**
• Understand ratios and rates, and solve problems involving proportional relationships (e.g., If it took 7 hours to mow 4 lawns, then at that rate, how many lawns could be mowed in 35 hours?).	• Divide fractions and solve related word problems (e.g., How wide is a rectangular strip of land with length 3⁄4 mile and area 1⁄2 square mile?). • Use positive and negative numbers together to describe quantities; understand the ordering and absolute values of positive and negative numbers.
Expressions and Equations	**Geometry**
• Work with variables and expressions by generalizing the way numbers work (e.g., When adding numbers, the order doesn't matter, so x + y = y + x; likewise, properties of addition and multiplication can be used to rewrite 24x + 18y as 6(4x + 3y), or y + y + y as 3y). • Write equations to solve word problems and describe relationships between quantities (e.g., The distance D traveled by a train in time T might be expressed by an equation D = 85T, where D is in miles and T is in hours.).	• Reason about relationships between shapes to determine area, surface area, and volume.
Statistics and Probability	
• Create graphical representations of data and reason about statistical distributions.	

Numbering of Standards

The following Mathematics Standards will be numbered continuously. The following ranges relate to the clusters found within Mathematics:

Ratios and Proportional Relationships	
Understand ratio concepts and use ratio reasoning to solve problems.	Standards 1-3
The Number System	
Apply and extend previous understandings of multiplication and division to divide fractions by fractions.	Standard 4
Compute fluently with multi-digit numbers and find common factors and multiples.	Standards 5-7
Apply and extend previous understandings of numbers to the system of rational numbers.	Standards 8-11
Expressions and Equations	
Apply and extend previous understandings of arithmetic to algebraic expressions.	Standards 12-15
Reason about and solve one-variable equations and inequalities.	Standards 16-19
Represent and analyze quantitative relationships between dependent and independent variables.	Standard 20
Geometry	
Solve real-world and mathematical problems involving area, surface area, and volume.	Standards 21-24
Statistics and Probability	
Develop understanding of statistical variability.	Standards 25-27
Summarize and describe distributions.	Standards 28-29

Ratios and Proportional Relationships

Cluster	Understand ratio concepts and use ratio reasoning to solve problems.
M.6.1	Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities. (e.g., “The ratio of wings to beaks in the bird house at the zoo was 2:1, because for every 2 wings there was 1 beak.” “For every vote candidate A received, candidate C received nearly three votes.”)
M.6.2	Understand the concept of a unit rate a/b associated with a ratio a:b with b ≠ 0, and use rate language in the context of a ratio relationship. (e.g., “This recipe has a ratio of 3 cups of flour to 4 cups of sugar, so there is 3/4 cup of flour for each cup of sugar.” “We paid $75 for 15 hamburgers, which is a rate of $5 per hamburger.”) Instructional Note: Expectations for unit rates in this grade are limited to non-complex fractions.
M.6.3	Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations. a. Make tables of equivalent ratios relating quantities with whole number measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios. b. Solve unit rate problems including those involving unit pricing and constant speed. (e.g., If it took 7 hours to mow 4 lawns, then at that rate, how many lawns could be mowed in 35 hours? At what rate were lawns being mowed?)

	c. Find a percent of a quantity as a rate per 100 (e.g., 30% of a quantity means 30/100 times the quantity); solve problems involving finding the whole, given a part and the percent. d. Use ratio reasoning to convert measurement units; manipulate and transform units appropriately when multiplying or dividing quantities.

The Number System

Cluster	Apply and extend previous understandings of multiplication and division to divide fractions by fractions.
M.6.4	Interpret and compute quotients of fractions and solve word problems involving division of fractions by fractions by using visual fraction models and equations to represent the problem. (e.g., Create a story context for (2/3) ÷ (3/4) and use a visual fraction model to show the quotient; use the relationship between multiplication and division to explain that (2/3) ÷ (3/4) = 8/9 because 3/4 of 8/9 is 2/3. (In general, (a/b) ÷ (c/d) = ad/bc.) How much chocolate will each person get if 3 people share 1/2 lb of chocolate equally? How many 3/4-cup servings are in 2/3 of a cup of yogurt? How wide is a rectangular strip of land with length 3/4 mi and area ½ square mi?)

Cluster	Compute fluently with multi-digit numbers and find common factors and multiples.
M.6.5	Fluently divide multi-digit numbers using the standard algorithm.
M.6.6	Fluently add, subtract, multiply and divide multi-digit decimals using the standard algorithm for each operation.
M.6.7	Find the greatest common factor of two whole numbers less than or equal to 100 and the least common multiple of two whole numbers less than or equal to 12. Use the distributive property to express a sum of two whole numbers 1–100 with a common factor as a multiple of a sum of two whole numbers with no common factor (e.g., express 36 + 8 as 4 (9 + 2)).

Cluster	Apply and extend previous understandings of numbers to the system of rational numbers.
M.6.8	Understand that positive and negative numbers are used together to describe quantities having opposite directions or values (e.g., temperature above/below zero, elevation above/below sea level, credits/debits, positive/negative electric charge); use positive and negative numbers to represent quantities in real-world contexts, explaining the meaning of 0 in each situation.
M.6.9	Understand a rational number as a point on the number line. Extend number line diagrams and coordinate axes familiar from previous grades to represent points on the line and in the plane with negative number coordinates. a. Recognize opposite signs of numbers as indicating locations on opposite sides of 0 on the number line; recognize that the opposite of the opposite of a number is the number itself, e.g., –(–3) = 3, and that 0 is its own opposite. b. Understand signs of numbers in ordered pairs as indicating locations in quadrants of the coordinate plane; recognize that when two ordered pairs differ only by signs, the locations of the points are related by reflections across one or both axes. c. Find and position integers and other rational numbers on a horizontal or vertical

	number line diagram; find and position pairs of integers and other rational numbers on a coordinate plane.
M.6.10	Understand ordering and absolute value of rational numbers. a. Interpret statements of inequality as statements about the relative position of two numbers on a number line diagram. (e.g., interpret –3 > –7 as a statement that –3 is located to the right of –7 on a number line oriented from left to right.) b. Write, interpret, and explain statements of order for rational numbers in real-world contexts (e.g., write $–3^{o}$ C > $–7^{o}$ C to express the fact that $–3^{o}$ C is warmer than $–7^{o}$ C). c. Understand the absolute value of a rational number as its distance from 0 on the number line; interpret absolute value as magnitude for a positive or negative quantity in a real-world situation. (e.g., for an account balance of –30 dollars, write |–30| = 30 to describe the size of the debt in dollars). d. Distinguish comparisons of absolute value from statements about order. (e.g., recognize that an account balance less than –30 dollars represents a debt greater than 30 dollars.)
M.6.11	Solve real-world and mathematical problems by graphing points in all four quadrants of the coordinate plane. Include use of coordinates and absolute value to find distances between points with the same first coordinate or the same second coordinate.

Expressions and Equations

Cluster	Apply and extend previous understandings of arithmetic to algebraic expressions.
M.6.12	Write and evaluate numerical expressions involving whole-number exponents.
M.6.13	Write, read and evaluate expressions in which letters stand for numbers. a. Write expressions that record operations with numbers and with letters standing for numbers. (e.g., Express the calculation, “Subtract y from 5” as 5 – y.) b. Identify parts of an expression using mathematical terms (sum, term, product, factor, quotient, coefficient); view one or more parts of an expression as a single entity. (e.g., Describe the expression 2 (8 + 7) as a product of two factors; view (8 + 7) as both a single entity and a sum of two terms.) c. Evaluate expressions at specific values of their variables. Include expressions that arise from formulas used in real-world problems. Perform arithmetic operations, including those involving whole number exponents, in the conventional order when there are no parentheses to specify a particular order: Order of Operations (e.g., use the formulas $V = s^3$ and $A = 6\ s^2$ to find the volume and surface area of a cube with sides of length s = 1/2).
M.6.14	Apply the properties of operations to generate equivalent expressions (e.g., apply the distributive property to the expression 3 (2 + x) to produce the equivalent expression 6 + 3x; apply the distributive property to the expression 24x + 18y to produce the equivalent expression 6 (4x + 3y); apply properties of operations to y + y + y to produce the equivalent expression 3y).
M.6.15	Identify when two expressions are equivalent; i.e., when the two expressions name the same number regardless of which value is substituted into them. (e.g., The expressions y + y + y and 3y are equivalent because they name the same number regardless of which number y stands for.)

Cluster	Reason about and solve one-variable equations and inequalities.
M.6.16	Understand solving an equation or inequality as a process of answering a question: which values from a specified set, if any, make the equation or inequality true? Use substitution to determine whether a given number in a specified set makes an equation or inequality true.
M.6.17	Use variables to represent numbers and write expressions when solving a real-world or mathematical problem; understand that a variable can represent an unknown number or depending on the purpose at hand, any number in a specified set.
M.6.18	Solve real-world and mathematical problems by writing and solving equations of the form $x + p = q$ and $px = q$ for cases in which p, q and x are all nonnegative rational numbers.
M.6.19	Write an inequality of the form $x > c$ or $x < c$ to represent a constraint or condition in a real-world or mathematical problem. Recognize that inequalities of the form $x > c$ or $x < c$ have infinitely many solutions; represent solutions of such inequalities on number line diagrams.

Cluster	Represent and analyze quantitative relationships between dependent and independent variables.
M.6.20	Use variables to represent two quantities in a real-world problem that change in relationship to one another; write an equation to express one quantity, thought of as the dependent variable, in terms of the other quantity, thought of as the independent variable. Analyze the relationship between the dependent and independent variables using graphs and tables, and relate these to the equation. (e.g., In a problem involving motion at constant speed, list and graph ordered pairs of distances and times, and write the equation $d = 65t$ to represent the relationship between distance and time.)

Geometry

Cluster	Solve real-world and mathematical problems involving area, surface area, and volume.
M.6.21	Find the area of right triangles, other triangles, special quadrilaterals and polygons by composing into rectangles or decomposing into triangles and other shapes; apply these techniques in the context of solving real-world and mathematical problems.
M.6.22	Find the volume of a right rectangular prism with fractional edge lengths by packing it with unit cubes of the appropriate unit fraction edge lengths and show that the volume is the same as would be found by multiplying the edge lengths of the prism. Apply the formulas $V = l\ w\ h$ and $V = B\ h$ to find volumes of right rectangular prisms with fractional edge lengths in the context of solving real-world and mathematical problems.
M.6.23	Draw polygons in the coordinate plane given coordinates for the vertices; use coordinates to find the length of a side joining points with the same first coordinate or the same second coordinate. Apply these techniques in the context of solving real-world and mathematical problems.
M.6.24	Represent three-dimensional figures using nets made up of rectangles and triangles, and use the nets to find the surface area of these figures. Apply these techniques in the context of solving real-world and mathematical problems.

Statistics and Probability

Cluster	Develop understanding of statistical variability.
M.6.25	Recognize a statistical question as one that anticipates variability in the data related to the question and accounts for it in the answers. (e.g., “How old am I?” is not a statistical question, but “How old are the students in my school?” is a statistical question because one anticipates variability in students’ ages.)
M.6.26	Through informal observation, understand that a set of data collected to answer a statistical question has a distribution which can be described by its center (mean/median), spread (range), and overall shape.
M.6.27	Recognize that a measure of center for a numerical data set summarizes all of its values with a single number.

Cluster	Summarize and describe distributions.
M.6.28	Display numerical data in plots on a number line, including dot plots, histograms and box plots.
M.6.29	Summarize numerical data sets in relation to their context, such as by: a. Reporting the number of observations. b. Describing the nature of the attribute under investigation, including how it was measured and its units of measurement. c. Giving quantitative measures of center (median and/or mean), as well as describing any overall pattern and any striking deviations from the overall pattern with reference to the context in which the data were gathered. d. Relating the choice of measures of center to the shape of the data distribution and the context in which the data were gathered.

Mathematics – Grade 7

All West Virginia teachers are responsible for classroom instruction that integrates content standards and mathematical habits of mind. Students in the seventh grade will focus on four critical areas: (1) developing understanding of and applying proportional relationships; (2) developing understanding of operations with rational numbers and working with expressions and linear equations; (3) solving problems involving scale drawings and informal geometric constructions and working with two- and three-dimensional shapes to solve problems involving area, surface area and volume; and (4) drawing inferences about populations based on samples. Mathematical habits of mind, which should be integrated in these content areas ,include: making sense of problems and persevering in solving them, reasoning abstractly and quantitatively; constructing viable arguments and critiquing the reasoning of others; modeling with mathematics; using appropriate tools strategically; attending to precision, looking for and making use of structure; and looking for and expressing regularity in repeated reasoning. Students in seventh grade will continue developing mathematical proficiency in a developmentally-appropriate progressions of standards. Continuing the skill progressions from sixth grade, the following chart represents the mathematical understandings that will be developed in seventh grade:

Ratios and Proportional Reasoning	The Number System
• Analyze proportional relationships (e.g., by graphing in the coordinate plane), and distinguish proportional relationships from other kinds of mathematical relationships (e.g., Buying 10 times as many items will cost you 10 times as much, but taking 10 times as many aspirin will not lower your fever 10 times as much.).	• Solve percent problems (e.g., tax, tips, and markups and markdowns). • Solve word problems that have a combination of whole numbers, fractions, and decimals (e.g., A woman making $25 per hour receives a 10% raise; she will make an additional $1/10$ of his or her salary an hour, or $2.50, for a new salary of $27.50.)
Expressions and Equations	**Geometry**
• Solve equations such as $1/2\ (x - 3) = 3/4$ quickly and accurately, and write equations of this kind to solve word problems.	• Solve problems involving scale drawings.
Statistics and Probability	
• Use statistics to draw inferences and make comparisons (e.g., deciding which candidate is likely to win an election based on a survey).	

Numbering of Standards

The following Mathematics Standards will be numbered continuously. The following ranges relate to the clusters found within Mathematics:

Ratios and Proportional Relationships	
Analyze proportional relationships and use them to solve real-world and mathematical problems.	Standards 1-3
The Number System	
Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers.	Standards 4-6

Expressions and Equations	
Use properties of operations to generate equivalent expressions.	Standards 7-8
Solve real-life and mathematical problems using numerical and algebraic expressions and equations.	Standards 9-10
Geometry	
Draw, construct and describe geometrical figures and describe the relationships between them.	Standards 11-13
Solve real-life and mathematical problems involving angle measure, area, surface area, and volume.	Standards 14-16
Statistics and Probability	
Use random sampling to draw inferences about a population.	Standards 17-18
Draw informal comparative inferences about two populations.	Standards 19-22
Investigate chance processes and develop, use, and evaluate probability models.	Standards 23-26

Ratios and Proportional Relationships

Cluster	Analyze proportional relationships and use them to solve real-world and mathematical problems.
M.7.1	Compute unit rates associated with ratios of fractions, including ratios of lengths, areas and other quantities measured in like or different units. (e.g., If a person walks 1/2 mile in each 1/4 hour, compute the unit rate as the complex fraction 1/2/1/4 miles per hour, equivalently 2 miles per hour.)
M.7.2	Recognize and represent proportional relationships between quantities. a. Decide whether two quantities are in a proportional relationship (e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin). b. Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams and verbal descriptions of proportional relationships. c. Represent proportional relationships by equations. (e.g., If total cost t is proportional to the number n of items purchased at a constant price p, the relationship between the total cost and the number of items can be expressed as t = pn.) d. Explain what a point (x,y) on the graph of a proportional relationship means in terms of the situation. Focus special attention on the points (0,0) and (1,r) where r is the unit rate.
M.7.3	Use proportional relationships to solve multistep ratio and percent problems (e.g., simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, and/or percent error).

The Number System

Cluster	**Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers.**
M.7.4	Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers; represent addition and subtraction on a horizontal or vertical number line diagram. a. Describe situations in which opposite quantities combine to make 0. (e.g., A hydrogen atom has 0 charge because its two constituents are oppositely charged.) b. Understand p + q as the number located a distance |q| from p, in the positive or negative direction, depending on whether q is positive or negative. (i.e., To add "p + q" on the number line, start at "0" and move to "p" then move |q| in the positive or negative direction depending on whether "q" is positive or negative.) Show that a number and its opposite have a sum of 0 (are additive inverses). Interpret sums of rational numbers by describing real-world contexts. c. Understand subtraction of rational numbers as adding the additive inverse, p – q = p + (–q). Show that the distance between two rational numbers on the number line is the absolute value of their difference and apply this principle in real-world contexts. d. Apply properties of operations as strategies to add and subtract rational numbers.
M.7.5	Apply and extend previous understandings of multiplication and division and of fractions to multiply and divide rational numbers. a. Understand that multiplication is extended from fractions to rational numbers by requiring that operations continue to satisfy the properties of operations, particularly the distributive property, leading to products such as (–1)(–1) = 1 and the rules for multiplying signed numbers. Interpret products of rational numbers by describing real-world contexts. b. Understand that integers can be divided, provided that the divisor is not zero, and every quotient of integers (with non-zero divisor) is a rational number. If p and q are integers, then –(p/q) = (–p)/q = p/(–q). Interpret quotients of rational numbers by describing real world contexts. c. Apply properties of operations as strategies to multiply and divide rational numbers. d. Convert a rational number to a decimal using long division; know that the decimal form of a rational number terminates in 0s or eventually repeats.
M.7.6	Solve real-world and mathematical problems involving the four operations with rational numbers. Instructional Note: Computations with rational numbers extend the rules for manipulating fractions to complex fractions.

Expressions and Equations

Cluster	**Use properties of operations to generate equivalent expressions.**
M.7.7	Apply properties of operations as strategies to add, subtract, factor and expand linear expressions with rational coefficients.
M.7.8	Understand that rewriting an expression in different forms in a problem context can shed light on the problem and how the quantities in it are related. (e.g., a + 0.05a = 1.05a means that "increase by 5%" is the same as "multiply by 1.05.")

Cluster	**Solve real-life and mathematical problems using numerical and algebraic expressions and equations.**
M.7.9	Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies. (e.g., If a woman making $25 an hour gets a 10% raise, she will make an additional 1/10 of her salary an hour, or $2.50, for a new salary of $27.50. If you want to place a towel bar 9 3/4 inches long in the center of a door that is 27 1/2 inches wide, you will need to place the bar about 9 inches from each edge; this estimate can be used as a check on the exact computation.)
M.7.10	Use variables to represent quantities in a real-world or mathematical problem and construct simple equations and inequalities to solve problems by reasoning about the quantities. a. Solve word problems leading to equations of the form px + q = r and p(x + q) = r, where p, q, and r are specific rational numbers. Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach. (e.g., The perimeter of a rectangle is 54 cm. Its length is 6 cm. What is its width? An arithmetic solution similar to “54 – 6 – 6 divided by 2” may be compared with the reasoning involved in solving the equation 2w – 12 = 54. An arithmetic solution similar to “54/2 – 6” may be compared with the reasoning involved in solving the equation 2(w – 6) = 54.) b. Solve word problems leading to inequalities of the form px + q > r or px + q < r, where p, q, and r are specific rational numbers. Graph the solution set of the inequality and interpret it in the context of the problem. (e.g., As a salesperson, you are paid $50 per week plus $3 per sale. This week you want your pay to be at least $100. Write an inequality for the number of sales you need to make, and describe the solutions.)

Geometry

Cluster	**Draw, construct and describe geometrical figures and describe the relationships between them.**
M.7.11	Solve problems involving scale drawings of geometric figures, including computing actual lengths and areas from a scale drawing and reproducing a scale drawing at a different scale.
M.7.12	Draw (freehand, with ruler and protractor, and with technology) geometric shapes with given conditions. Focus on constructing triangles from three measures of angles or sides, noticing when the conditions determine a unique triangle, more than one triangle, or no triangle.
M.7.13	Describe the two-dimensional figures that result from slicing three-dimensional figures, as in plane sections of right rectangular prisms and right rectangular pyramids.

Cluster	**Solve real-life and mathematical problems involving angle measure, area, surface area, and volume.**

M.7.14	Know the formulas for the area and circumference of a circle and use them to solve problems; give an informal derivation of the relationship between the circumference and area of a circle.
M.7.15	Use facts about supplementary, complementary, vertical, and adjacent angles in a multi-step problem to write and solve simple equations for an unknown angle in a figure.
M.7.16	Solve real-world and mathematical problems involving area, volume and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms.

Statistics and Probability

Cluster	Use random sampling to draw inferences about a population.
M.7.17	Understand that statistics can be used to gain information about a population by examining a sample of the population; generalizations about a population from a sample are valid only if the sample is representative of that population. Understand that random sampling tends to produce representative samples and support valid inferences.
M.7.18	Use data from a random sample to draw inferences about a population with an unknown characteristic of interest. Generate multiple samples (or simulated samples) of the same size to gauge the variation in estimates or predictions. (e.g., Estimate the mean word length in a book by randomly sampling words from the book; predict the winner of a school election based on randomly sampled survey data. Gauge how far off the estimate or prediction might be.)

Cluster	Draw informal comparative inferences about two populations.
M.7.19	Recognize that a measure of center for a numerical data set summarizes all of its values with a single number, while a measure of variation describes how its values vary with a single number.
M.7.20	Summarize numerical data sets in relation to their context, such as by: a. Reporting the number of observations. b. Describing the nature of the attribute under investigation, including how it was measured and its units of measurement. c. Giving quantitative measures of center (median and/or mean) and variability (interquartile range and/or mean absolute deviation), as well as describing any overall pattern and any striking deviations from the overall pattern with reference to the context in which the data were gathered. Relating the choice of measures of center and variability to the shape of the data distribution and the context in which the data were gathered.
M.7.21	Informally assess the degree of visual overlap of two numerical data distributions with similar variabilities, measuring the difference between the centers by expressing it as a multiple of a measure of variability. (e.g., The mean height of players on the basketball team is 10 cm greater than the mean height of players on the soccer team, about twice the variability (mean absolute deviation) on either team; on a dot plot, the separation between the two distributions of heights is noticeable.)
M.7.22	Use measures of center and measures of variability for numerical data from random samples to draw informal comparative inferences about two populations. (e.g., Decide whether the words in a chapter of a seventh-grade science book are generally longer than the words in a chapter of a fourth-grade science book.)

Cluster	Investigate chance processes and develop, use, and evaluate probability models.
M.7.23	Understand that the probability of a chance event is a number between 0 and 1 that expresses the likelihood of the event occurring. Larger numbers indicate greater likelihood. A probability near 0 indicates an unlikely event, a probability around 1/2 indicates an event that is neither unlikely nor likely and a probability near 1 indicates a likely event.
M.7.24	Approximate the probability of a chance event by collecting data on the chance process that produces it and observing its long-run relative frequency, and predict the approximate relative frequency given the probability. (e.g., When rolling a number cube 600 times, predict that a 3 or 6 would be rolled roughly 200 times, but probably not exactly 200 times.)
M.7.25	Develop a probability model and use it to find probabilities of events. Compare probabilities from a model to observed frequencies; if the agreement is not good, explain possible sources of the discrepancy. a. Develop a uniform probability model by assigning equal probability to all outcomes, and use the model to determine probabilities of events. (e.g., If a student is selected at random from a class, find the probability that Jane will be selected and the probability that a girl will be selected.) b. Develop a probability model (which may not be uniform) by observing frequencies in data generated from a chance process. (e.g., Find the approximate probability that a spinning penny will land heads up or that a tossed paper cup will land open-end down. Do the outcomes for the spinning penny appear to be equally likely based on the observed frequencies?)
M.7.26	Find probabilities of compound events using organized lists, tables, tree diagrams, and simulation. a. Understand that, just as with simple events, the probability of a compound event is the fraction of outcomes in the sample space for which the compound event occurs. b. Represent sample spaces for compound events using methods such as organized lists, tables and tree diagrams. For an event described in everyday language (e.g., “rolling double sixes”), identify the outcomes in the sample space which compose the event. c. Design and use a simulation to generate frequencies for compound events. (e.g., Use random digits as a simulation tool to approximate the answer to the question: If 40% of donors have type A blood, what is the probability that it will take at least 4 donors to find one with type A blood?)

Mathematics – Grade 8

All West Virginia teachers are responsible for classroom instruction that integrates content standards and mathematical habits of mind. Students in the eighth grade will focus on three critical areas: 1) formulating and reasoning about expressions and equations, including modeling an association in bivariate data with a linear equation, and solving linear equations and systems of linear equations; (2) grasping the concept of a function and using functions to describe quantitative relationships; (3) analyzing two- and three-dimensional space and figures using distance, angle, similarity and congruence and understanding and applying the Pythagorean Theorem. Mathematical habits of mind, which should be integrated in these content areas, include: making sense of problems and persevering in solving them, reasoning abstractly and quantitatively; constructing viable arguments and critiquing the reasoning of others; modeling with mathematics; using appropriate tools strategically; attending to precision, looking for and making use of structure; and looking for and expressing regularity in repeated reasoning. Students in eighth grade will continue developing mathematical proficiency in a developmentally-appropriate progressions of standards. Continuing the skill progressions from seventh grade, the following chart represents the mathematical understandings that will be developed in eighth grade:

The Number System	Expressions and Equations
• Understand that every number has a decimal expansion and use these to compare the size of irrational numbers.	• Work with positive and negative exponents, square root and cube root symbols, and scientific notation (e.g., Evaluate $\sqrt{36} + 64$; estimate world population as 7×10^9). • Solve linear equations (e.g., $-x + 5(x + 1/3) = 2x - 8$); solve pairs of linear equations (e.g., $x + 6y = -1$ and $2x - 2y = 12$); and write equations to solve related word problems.
Functions	**Geometry**
• Understand slope, and relating linear equations in two variables to lines in the coordinate plane. • Understand functions as rules that assign a unique output number to each input number; use linear functions to model relationships. **Statistics and Probability** • Analyze statistical relationships by using a best-fit line (a straight line that models an association between two quantities).	• Understand congruence and similarity using physical models, transparencies, or geometry software (e.g., Given two congruent figures, show how to obtain one from the other by a sequence of rotations, translations, and/or reflections).

Numbering of Standards

The following Mathematics Standards will be numbered continuously. The following ranges relate to the clusters found within Mathematics:

The Number System	
Know that there are numbers that are not rational, and approximate them by rational	Standards 1-2

numbers.	
Expressions and Equations	
Work with radicals and integer exponents.	Standards 3-6
Understand the connections between proportional relationships, lines, and linear equations.	Standards 7-8
Analyze and solve linear equations and pairs of simultaneous linear equations.	Standards 9-10
Functions	
Define, evaluate, and compare functions.	Standards 11-13
Use functions to model relationships between quantities.	Standards 14-15
Geometry	
Understand congruence and similarity using physical models, transparencies, or geometry software.	Standards 16-20
Understand and apply the Pythagorean Theorem.	Standards 21-23
Solve real-world and mathematical problems involving volume of cylinders, cones, and spheres.	Standard 24
Statistics and Probability	
Investigate patterns of association in bivariate data.	Standards 25-28

The Number System

Cluster	Know that there are numbers that are not rational, and approximate them by rational numbers.
M.8.1	Know that numbers that are not rational are called irrational. Understand informally that every number has a decimal expansion; for rational numbers show that the decimal expansion repeats eventually and convert a decimal expansion which repeats eventually into a rational number. Instructional Note: A decimal expansion that repeats the digit 0 is often referred to as a “terminating decimal.”
M.8.2	Use rational approximations of irrational numbers to compare the size of irrational numbers, locate them approximately on a number line diagram and estimate the value of expressions such as π^2. (e.g., By truncating the decimal expansion of √2, show that √2 is between 1 and 2, then between 1.4 and 1.5, and explain how to continue on to get better approximations.)

Expressions and Equations

Cluster	Work with radicals and integer exponents.
M.8.3	Know and apply the properties of integer exponents to generate equivalent numerical expressions. (e.g., $3^2 \times 3^{-5} = 3^{-3} = 1/3^3 = 1/27$.)
M.8.4	Use square root and cube root symbols to represent solutions to equations of the form $x^2 = p$ and $x^3 = p$, where p is a positive rational number. Evaluate square roots of small perfect squares and cube roots of small perfect cubes. Know that √2 is irrational.

M.8.5	Use numbers expressed in the form of a single digit times an integer power of 10 to estimate very large or very small quantities, and to express how many times as much one is than the other. (e.g., Estimate the population of the United States as 3×10^8 and the population of the world as 7×10^9, and determine that the world population is more than 20 times larger.)
M.8.6	Perform operations with numbers expressed in scientific notation, including problems where both decimal and scientific notation are used. Use scientific notation and choose units of appropriate size for measurements of very large or very small quantities. (e.g., Use millimeters per year for seafloor spreading.) Interpret scientific notation that has been generated by technology.

Cluster	**Understand the connections between proportional relationships, lines, and linear equations.**
M.8.7	Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways. (e.g., Compare a distance-time graph to a distance-time equation to determine which of two moving objects has greater speed.)
M.8.8	Use similar triangles to explain why the slope m is the same between any two distinct points on a non-vertical line in the coordinate plane; derive the equation $y = mx$ for a line through the origin and the equation $y = mx + b$ for a line intercepting the vertical axis at b.

Cluster	**Analyze and solve linear equations and pairs of simultaneous linear equations.**
M.8.9	Solve linear equations in one variable. a. Give examples of linear equations in one variable with one solution, infinitely many solutions or no solutions. Show which of these possibilities is the case by successively transforming the given equation into simpler forms, until an equivalent equation of the form $x = a$, $a = a$, or $a = b$ results (where a and b are different numbers). b. Solve linear equations with rational number coefficients, including equations whose solutions require expanding expressions using the distributive property and collecting like terms.
M.8.10	Analyze and solve pairs of simultaneous linear equations. a. Understand that solutions to a system of two linear equations in two variables correspond to points of intersection of their graphs, because points of intersection satisfy both equations simultaneously. b. Solve systems of two linear equations in two variables algebraically and estimate solutions by graphing the equations. Solve simple cases by inspection. (e.g., $3x + 2y = 5$ and $3x + 2y = 6$ have no solution because $3x + 2y$ cannot simultaneously be 5 and 6.) c. Solve real-world and mathematical problems leading to two linear equations in two variables. (e.g., Given coordinates for two pairs of points, determine whether the line through the first pair of points intersects the line through the second pair.)

Functions

Cluster	**Define, evaluate, and compare functions.**
M.8.11	Understand that a function is a rule that assigns to each input exactly one output. The graph

	of a function is the set of ordered pairs consisting of an input and the corresponding output. Instructional Note: Function notation is not required in grade 8.
M.8.12	Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). (e.g., Given a linear function represented by a table of values and a linear function represented by an algebraic expression, determine which function has the greater rate of change.)
M.8.13	Interpret the equation $y = mx + b$ as defining a linear function, whose graph is a straight line; give examples of functions that are not linear. (e.g., The function $A = s^2$ giving the area of a square as a function of its side length is not linear because its graph contains the points (1,1), (2,4) and (3,9), which are not on a straight line.)

Cluster	Use functions to model relationships between quantities
M.8.14	Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (x, y) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values.
M.8.15	Describe qualitatively the functional relationship between two quantities by analyzing a graph (e.g., where the function is increasing or decreasing, linear or nonlinear). Sketch a graph that exhibits the qualitative features of a function that has been described verbally.

Geometry

Cluster	Understand congruence and similarity using physical models, transparencies, or geometry software.
M.8.16	Verify experimentally the properties of rotations, reflections and translations: a. Lines are taken to lines, and line segments to line segments of the same length. b. Angles are taken to angles of the same measure. c. Parallel lines are taken to parallel lines.
M.8.17	Understand that a two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of rotations, reflections and translations; given two congruent figures, describe a sequence that exhibits the congruence between them.
M.8.18	Describe the effect of dilations, translations, rotations and reflections on two-dimensional figures using coordinates.
M.8.19	Understand that a two-dimensional figure is similar to another if the second can be obtained from the first by a sequence of rotations, reflections, translations and dilations; given two similar two dimensional figures, describe a sequence that exhibits the similarity between them.
M.8.20	Use informal arguments to establish facts about the angle sum and exterior angle of triangles, about the angles created when parallel lines are cut by a transversal, and the angle-angle criterion for similarity of triangles. (e.g., Arrange three copies of the same triangle so that the sum of the three angles appears to form a line, and give an argument in terms of transversals why this is so.)

Cluster	Understand and apply the Pythagorean Theorem.
M.8.21	Explain a proof of the Pythagorean Theorem and its converse.
M.8.22	Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in

	real-world and mathematical problems in two and three dimensions.
M.8.23	Apply the Pythagorean Theorem to find the distance between two points in a coordinate system.

Cluster	Solve real-world and mathematical problems involving volume of cylinders, cones, and spheres.
M.8.24	Know the formulas for the volumes of cones, cylinders and spheres and use them to solve real-world and mathematical problems.

Statistics and Probability

Cluster	Investigate patterns of association in bivariate data.
M.8.25	Construct and interpret scatter plots for bivariate measurement data to investigate patterns of association between two quantities. Describe patterns such as clustering, outliers, positive or negative association, linear association and nonlinear association.
M.8.26	Know that straight lines are widely used to model relationships between two quantitative variables. For scatter plots that suggest a linear association, informally fit a straight line and informally assess the model fit by judging the closeness of the data points to the line.
M.8.27	Use the equation of a linear model to solve problems in the context of bivariate measurement data, interpreting the slope and intercept. (e.g., In a linear model for a biology experiment, interpret a slope of 1.5 cm/hr as meaning that an additional hour of sunlight each day is associated with an additional 1.5 cm in mature plant height.)
M.8.28	Understand that patterns of association can also be seen in bivariate categorical data by displaying frequencies and relative frequencies in a two-way table. Construct and interpret a two-way table summarizing data on two categorical variables collected from the same subjects. Use relative frequencies calculated for rows or columns to describe possible association between the two variables. (e.g., Collect data from students in your class on whether or not they have a curfew on school nights and whether or not they have assigned chores at home. Is there evidence that those who have a curfew also tend to have chores?)

High School Mathematics

At the high school level, the standards are organized by conceptual category (number and quantity, algebra, functions, geometry, modeling and probability and statistics), showing the body of knowledge students should learn in each category to be college- and career-ready and to be prepared to study more advanced mathematics. There are two distinct course sequence pathways of the high school standards for the mathematics progression in grades 9-11:

- The Integrated Pathway with a course sequence of Math I, Math II, and Math III, each of which includes number, algebra, geometry, probability and statistics; and
- The Traditional Pathway with a course sequence of Algebra I, Geometry, and Algebra II, with some data, probability and statistics included in each course.

Each pathway organizes the identical standards into courses that provide a strong foundation for post-secondary success. As a result, the mathematics standards identified in Math I, Math II and Math III are identical to the standards identified in Algebra I, Geometry and Algebra II. The content is simply grouped differently among the three years. Local Education Agencies (LEA) must choose to implement either the Integrated or Traditional Pathway. Regardless of the pathway chosen for grades 9-11, the fourth course options for all students are the same.

INTEGRATED PATHWAY

Mathematics – 8th Grade High School Mathematics I

All West Virginia teachers are responsible for classroom instruction that integrates content standards and mathematical habits of mind. Students in this course will focus on six critical units that deepen and extend understanding of linear and exponential relationships by contrasting them with each other and by applying linear models to data that exhibit a linear trend. Students explore the role of rigid motions in congruence and similarity, are introduced to the Pythagorean Theorem, and examine volume relationships of cones, cylinders and spheres. Students in 8th Grade Mathematics 1 use properties and theorems involving congruent figures to deepen and extend understanding of geometric knowledge from prior grades and develop connections between the algebraic and geometric ideas studied. Mathematical habits of mind, which, include: making sense of problems and persevering in solving them, reasoning abstractly and quantitatively; constructing viable arguments and critiquing the reasoning of others; modeling with mathematics; using appropriate tools strategically; attending to precision, looking for and making use of structure; and looking for and expressing regularity in repeated reasoning. Students will continue developing mathematical proficiency in a developmentally-appropriate progressions of standards. Continuing the skill progressions from seventh grade, the following chart represents the mathematical understandings that will be developed:

Relationships between Quantities	Linear and Exponential Relationships
• Solve problems with a wide range of units and solve problems by thinking about units. (e.g., The Trans Alaska Pipeline System is 800 miles long and cost $8 billion to build. Divide one of these numbers by the other. What is the meaning of the answer? Greenland has a population of 56,700 and a land area of 2,175,600 square kilometers. By what factor is the population density of the United States, 80 persons per square mile, larger than the population density of Greenland?)	• Understand contextual relationships of variables and constants. (e.g., Annie is picking apples with her sister. The number of apples in her basket is described by n = 22t + 12, where t is the number of minutes Annie spends picking apples. What do the numbers 22 and 12 tell you about Annie's apple picking?)
Reasoning with Equations	**Descriptive Statistics**
• Translate between various forms of linear equations. (e.g., The perimeter of a rectangle is given by P = 2W + 2L. Solve for W and restate in words the meaning of this new formula in terms of the meaning of the other variables.). • Explore systems of equations, find and interpret their solutions. (e.g., The high school is putting on the musical Footloose. The auditorium has 300 seats. Student tickets are $3 and adult tickets are $5. The royalty for the musical is $1300. What combination of student and adult tickets do you need to fill the house and pay the royalty? How could	• Use linear regression techniques to describe the relationship between quantities and assess the fit of the model. (e.g., Use the high school and university grades for 250 students to create a model that can be used to predict a student's university GPA based on his high school GPA.)

you change the price of tickets so more students can go?)	
Congruence, Proof, and Constructions	**Connecting Algebra and Geometry through Coordinates**
• Given a transformation, work backwards to discover the sequence that led to the transformation. • Given two quadrilaterals that are reflections of each other, find the line of that reflection.	• Use a rectangular coordinate system and build on understanding of the Pythagorean Theorem to find distances. (e.g., Find the area and perimeter of a real-world shape using a coordinate grid and Google Earth.) • Analyze the triangles and quadrilaterals on the coordinate plane to determine their properties. (e.g., Determine whether a given quadrilateral is a rectangle.)

Numbering of Standards

The following Mathematics Standards will be numbered continuously. The following ranges relate to the clusters found within Mathematics:

Relationships between Quantities	
Reason quantitatively and use units to solve problems.	Standards 1-3
Interpret the structure of expressions.	Standard 4
Create equations that describe numbers or relationships.	Standards 5-8
Linear and Exponential Relationships	
Represent and solve equations and inequalities graphically.	Standards 9-11
Define, evaluate, and compare functions.	Standards 12-14
Understand the concept of a function and use function notation.	Standards 15-17
Use functions to model relationships between quantities.	Standards 18-19
Interpret functions that arise in applications in terms of a context.	Standards 20-22
Analyze functions using different representations.	Standards 23-24
Build a function that models a relationship between two quantities.	Standards 25-26
Build new functions from existing functions.	Standard 27
Construct and compare linear, quadratic, and exponential models and solve problems.	Standards 28-30
Interpret expressions for functions in terms of the situation they model.	Standard 31
Reasoning with Equations	
Understand solving equations as a process of reasoning and explain the reasoning.	Standard 32
Solve equations and inequalities in one variable.	Standard 33

Analyze and solve linear equations and pairs of simultaneous linear equations.	Standard 34
Solve systems of equations.	Standards 35-36
Descriptive Statistics	
Summarize, represent, and interpret data on a single count or measurement variable.	Standards 37-39
Investigate patterns of association in bivariate data.	Standards 40-43
Summarize, represent, and interpret data on two categorical and quantitative variables.	Standards 44-45
Interpret linear models.	Standards 46-48
Congruence, Proof, and Constructions	
Experiment with transformations in the plane.	Standards 49-53
Understand congruence in terms of rigid motions.	Standards 54-56
Make geometric constructions.	Standards 57-58
Understand and apply the Pythagorean theorem.	Standards 59-61
Connecting Algebra and Geometry through Coordinates	
Use coordinates to prove simple geometric theorems algebraically.	Standards 62-64

Relationships between Quantities

Cluster	Reason quantitatively and use units to solve problems.
M.1HS8.1	Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.
M.1HS8.2	Define appropriate quantities for the purpose of descriptive modeling. Instructional Note: Working with quantities and the relationships between them provides grounding for work with expressions, equations, and functions.
M.1HS8.3	Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.

Cluster	Interpret the structure of expressions.
M.1HS8.4	Interpret expressions that represent a quantity in terms of its context. a. Interpret parts of an expression, such as terms, factors, and coefficients. b. Interpret complicated expressions by viewing one or more of their parts as a single entity. For example, interpret $P(1 + r)^n$ as the product of P and a factor not depending on P. Instructional Note: Limit to linear expressions and to exponential expressions with integer exponents.

Cluster	Create equations that describe numbers or relationships.
M.1HS8.5	Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions and simple rational and exponential functions. Instructional Note: Limit to linear and exponential equations and in the case of exponential equations, limit to situations requiring evaluation of exponential functions at integer inputs.

M.1HS8.6	Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales. Instructional Note: Limit to linear and exponential equations and in the case of exponential equations, limit to situations requiring evaluation of exponential functions at integer inputs.
M.1HS8.7	Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context. (e.g., Represent inequalities describing nutritional and cost constraints on combinations of different foods.) Instructional Note: Limit to linear equations and inequalities.
M.1HS8.8	Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. (e.g., Rearrange Ohm's law V = IR to highlight resistance R.) Instructional Note: Limit to formulas with a linear focus.

Linear and Exponential Relationships

Cluster	**Represent and solve equations and inequalities graphically.**
M.1HS8.9	Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line). Instructional Note: Focus on linear and exponential equations and be able to adapt and apply that learning to other types of equations in future courses.
M.1HS8.10	Explain why the x-coordinates of the points where the graphs of the equations y = f(x) and y = g(x) intersect are the solutions of the equation f(x) = g(x); find the solutions approximately, (e.g., using technology to graph the functions, make tables of values, or find successive approximations). Include cases where f(x) and/or g(x) are linear, polynomial, rational, absolute value exponential, and logarithmic functions. Instructional Note: Focus on cases where f(x) and g(x) are linear or exponential.
M.1HS8.11	Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality) and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.

Cluster	**Define, evaluate, and compare functions.** Instructional Note: While this content is likely subsumed by M.1HS8.12-14 and M.1HS8.26a, it could be used for scaffolding instruction to the more sophisticated content found there.
M.1HS8.12	Understand that a function is a rule that assigns to each input exactly one output. The graph of a function is the set of ordered pairs consisting of an input and the corresponding output.
M.1HS8.13	Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). (e.g., Given a linear function represented by a table of values and a linear function represented by an algebraic expression, determine which function has the greater rate of change.)
M.1HS8.14	Interpret the equation y = mx + b as defining a linear function, whose graph is a straight line; give examples of functions that are not linear. (e.g., The function $A = s^2$ giving the area of a square as a function of its side length is not linear because its graph contains the points (1,1), (2,4) and (3,9), which are not on a straight line.)

Cluster	**Understand the concept of a function and use function notation.**
M.1HS8.15	Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then f(x) denotes the output of f corresponding to the input x. The graph of f is the graph of the equation y = f(x). Instructional Note: Students should experience a variety of types of situations modeled by functions. Detailed analysis of any particular class of function at this stage is not advised. Students should apply these concepts throughout their future mathematics courses. Constrain to linear functions and exponential functions having integral domains.
M.1HS8.16	Use function notation, evaluate functions for inputs in their domains and interpret statements that use function notation in terms of a context. Instructional Note: Students should experience a variety of types of situations modeled by functions. Detailed analysis of any particular class of function at this stage is not advised. Students should apply these concepts throughout their future mathematics courses. Constrain to linear functions and exponential functions having integral domains.
M.1HS8.17	Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. (e.g., The Fibonacci sequence is defined recursively by f(0) = f(1) = 1, f(n+1) = f(n)+ f(n-1) for n ≥ 1.) Instructional Note: Students should experience a variety of types of situations modeled by functions. Detailed analysis of any particular class of function at this stage is not advised. Students should apply these concepts throughout their future mathematics courses. Constrain to linear functions and exponential functions having integral domains. Draw connection to M.1HS8.26, which requires students to write arithmetic and geometric sequences.

Cluster	**Use functions to model relationships between quantities.** Instructional Note: While this content is likely subsumed by M.1HS8.20 and M.1HS8.25a, it could be used for scaffolding instruction to the more sophisticated content found there.
M.1HS8.18	Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (x, y) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values.
M.1HS8.19	Describe qualitatively the functional relationship between two quantities by analyzing a graph (e.g., where the function is increasing or decreasing, linear or nonlinear). Sketch a graph that exhibits the qualitative features of a function that has been described verbally.

Cluster	**Interpret functions that arise in applications in terms of a context.**
M.1HS8.20	For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive or negative; relative maximums and minimums; symmetries; end behavior; and periodicity. Instructional Note: Focus on linear and exponential functions.

M.1HS8.21	Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. (e.g., If the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.) Instructional Note: Focus on linear and exponential functions.
M.1HS8.22	Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph. Instructional Note: Focus on linear functions and intervals for exponential functions whose domain is a subset of the integers. Mathematics II and III will address other function types.

Cluster	**Analyze functions using different representations.**
M.1HS8.23	Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. a. Graph linear and quadratic functions and show intercepts, maxima, and minima. b. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude. Instructional Note: Focus on linear and exponential functions. Include comparisons of two functions presented algebraically. For example, compare the growth of two linear functions, or two exponential functions such as $y = 3^n$ and $y = 100 \times 2^n$.
M.1HS8.24	Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). (e.g., Given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.) Instructional Note: Focus on linear and exponential functions. Include comparisons of two functions presented algebraically. For example, compare the growth of two linear functions, or two exponential functions such as $y = 3^n$ and $y = 100 \times 2^n$.

Cluster	**Build a function that models a relationship between two quantities.**
M.1HS8.25	Write a function that describes a relationship between two quantities. a. Determine an explicit expression, a recursive process or steps for calculation from a context. b. Combine standard function types using arithmetic operations. (e.g., Build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.) Instructional Note: Limit to linear and exponential functions.
M.1HS8.26	Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms. Instructional Note: Limit to linear and exponential functions. Connect arithmetic sequences to linear functions and geometric sequences to exponential functions.

Cluster	**Build new functions from existing functions.**
M.1HS8.27	Identify the effect on the graph of replacing f(x) by f(x) + k, k f(x), f(kx), and f(x + k) for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using

	technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them. Instructional Note: Focus on vertical translations of graphs of linear and exponential functions. Relate the vertical translation of a linear function to its y-intercept. While applying other transformations to a linear graph is appropriate at this level, it may be difficult for students to identify or distinguish between the effects of the other transformations included in this standard.

Cluster	**Construct and compare linear, quadratic, and exponential models and solve problems.**
M.1HS8.28	Distinguish between situations that can be modeled with linear functions and with exponential functions. a. Prove that linear functions grow by equal differences over equal intervals; exponential functions grow by equal factors over equal intervals. b. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another. c. Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.
M.1HS8.29	Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).
M.1HS8.30	Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function. Instructional Note: Limit to comparisons between exponential and linear models.

Cluster	**Interpret expressions for functions in terms of the situation they model.**
M.1HS8.31	Interpret the parameters in a linear or exponential function in terms of a context. Instructional Note: Limit exponential functions to those of the form $f(x) = b^x + k$.

Reasoning with Equations

Cluster	**Understand solving equations as a process of reasoning and explain the reasoning.**
M.1HS8.32	Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method. Instructional Note: Students should focus on linear equations and be able to extend and apply their reasoning to other types of equations in future courses. Students will solve exponential equations with logarithms in Mathematics III.

Cluster	**Solve equations and inequalities in one variable.**
M.1HS8.33	Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters. Instructional Note: Extend earlier work with solving linear equations to solving linear inequalities in one variable and to solving literal equations that are linear in the variable being solved for. Include simple exponential equations that rely only on application of the laws of exponents, such as $5^x = 125$ or $2^x = 1/16$.

Cluster	**Analyze and solve linear equations and pairs of simultaneous linear equations.**
M.1HS8.34	Analyze and solve pairs of simultaneous linear equations. a. Understand that solutions to a system of two linear equations in two variables correspond to points of intersection of their graphs, because points of intersection satisfy both equations simultaneously. b. Solve systems of two linear equations in two variables algebraically, and estimate solutions by graphing the equations. Solve simple cases by inspection. For example, $3x + 2y = 5$ and $3x + 2y = 6$ have no solution because $3x + 2y$ cannot simultaneously be 5 and 6. c. Solve real-world and mathematical problems leading to two linear equations in two variables. For example, given coordinates for two pairs of points, determine whether the line through the first pair of points intersects the line through the second pair. Instructional Note: While this content is likely subsumed by M.1HS8.33, 35, and 36, it could be used for scaffolding instruction to the more sophisticated content found there.

Cluster	**Solve systems of equations.**
M.1HS8.35	Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions. Instructional Note: Include cases where two equations describe the same line (yielding infinitely many solutions) and cases where two equations describe parallel lines (yielding no solution).
M.1HS8.36	Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables. Instructional Note: Include cases where two equations describe the same line (yielding infinitely many solutions) and cases where two equations describe parallel lines (yielding no solution).

Descriptive Statistics

Cluster	**Summarize, represent, and interpret data on a single count or measurement variable.**
M.1HS8.37	Represent data with plots on the real number line (dot plots, histograms, and box plots).
M.1HS8.38	Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets. Instructional Note: In grades 6 – 7, students describe center and spread in a data distribution. Here they choose a summary statistic appropriate to the characteristics of the data distribution, such as the shape of the distribution or the existence of extreme data points.
M.1HS8.39	Interpret differences in shape, center and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers). Instructional Note: In grades 6 – 7, students describe center and spread in a data distribution. Here they choose a summary statistic appropriate to the characteristics of the data distribution, such as the shape of the distribution or the existence of extreme data points.

Cluster	**Investigate patterns of association in bivariate data.**

	Instructional Note: While this content is likely subsumed by M.1HS8.45-48, it could be used for scaffolding instruction to the more sophisticated content found there.
M.1HS8.40	Construct and interpret scatter plots for bivariate measurement data to investigate patterns of association between two quantities. Describe patterns such as clustering, outliers, positive or negative association, linear association and nonlinear association.
M.1HS8.41	Know that straight lines are widely used to model relationships between two quantitative variables. For scatter plots that suggest a linear association, informally fit a straight line and informally assess the model fit by judging the closeness of the data points to the line.
M.1HS8.42	Use the equation of a linear model to solve problems in the context of bivariate measurement data, interpreting the slope and intercept. (e.g., In a linear model for a biology experiment, interpret a slope of 1.5 cm/hr as meaning that an additional hour of sunlight each day is associated with an additional 1.5 cm in mature plant height.)
M.1HS8.43	Understand that patterns of association can also be seen in bivariate categorical data by displaying frequencies and relative frequencies in a two-way table. Construct and interpret a two-way table summarizing data on two categorical variables collected from the same subjects. Use relative frequencies calculated for rows or columns to describe possible association between the two variables. (e.g., Collect data from students in your class on whether or not they have a curfew on school nights and whether or not they have assigned chores at home. Is there evidence that those who have a curfew also tend to have chores?)

Cluster	**Summarize, represent, and interpret data on two categorical and quantitative variables.**
M.1HS8.44	Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal and conditional relative frequencies). Recognize possible associations and trends in the data.
M.1HS8.45	Represent data on two quantitative variables on a scatter plot, and describe how the variables are related. a. Fit a function to the data; use functions fitted to data to solve problems in the context of the data. Use given functions or choose a function suggested by the context. Emphasize linear and exponential models. b. Informally assess the fit of a function by plotting and analyzing residuals. (Focus should be on situations for which linear models are appropriate.) c. Fit a linear function for scatter plots that suggest a linear association. Instructional Note: Students take a more sophisticated look at using a linear function to model the relationship between two numerical variables. In addition to fitting a line to data, students assess how well the model fits by analyzing residuals.

Cluster	**Interpret linear models.**
M.1HS8.46	Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data. Instructional Note: Build on students' work with linear relationships and introduce the correlation coefficient. The focus here is on the computation and interpretation of the correlation coefficient as a measure of how well the data fit the relationship.
M.1HS8.47	Compute (using technology) and interpret the correlation coefficient of a linear fit.

	Instructional Note: Build on students' work with linear relationships and introduce the correlation coefficient. The focus here is on the computation and interpretation of the correlation coefficient as a measure of how well the data fit the relationship.
M.1HS8.48	Distinguish between correlation and causation. Instructional Note: The important distinction between a statistical relationship and a cause-and-effect relationship arises here.

Congruence, Proof, and Constructions

Cluster	Experiment with transformations in the plane.
M.1HS8.49	Know precise definitions of angle, circle, perpendicular line, parallel line and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.
M.1HS8.50	Represent transformations in the plane using, example, transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not (e.g., translation versus horizontal stretch). Instructional Note: Build on student experience with rigid motions from earlier grades. Point out the basis of rigid motions in geometric concepts, (e.g., translations move points a specified distance along a line parallel to a specified line; rotations move objects along a circular arc with a specified center through a specified angle).
M.1HS8.51	Given a rectangle, parallelogram, trapezoid or regular polygon, describe the rotations and reflections that carry it onto itself. Instructional Note: Build on student experience with rigid motions from earlier grades. Point out the basis of rigid motions in geometric concepts, (e.g., translations move points a specified distance along a line parallel to a specified line; rotations move objects along a circular arc with a specified center through a specified angle).
M.1HS8.52	Develop definitions of rotations, reflections and translations in terms of angles, circles, perpendicular lines, parallel lines and line segments. Instructional Note: Build on student experience with rigid motions from earlier grades. Point out the basis of rigid motions in geometric concepts, (e.g., translations move points a specified distance along a line parallel to a specified line; rotations move objects along a circular arc with a specified center through a specified angle).
M.1HS8.53	Given a geometric figure and a rotation, reflection or translation draw the transformed figure using, e.g., graph paper, tracing paper or geometry software. Specify a sequence of transformations that will carry a given figure onto another. Instructional Note: Build on student experience with rigid motions from earlier grades. Point out the basis of rigid motions in geometric concepts, (e.g., translations move points a specified distance along a line parallel to a specified line; rotations move objects along a circular arc with a specified center through a specified angle).

Cluster	Understand congruence in terms of rigid motions.
M.1HS8.54	Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent. Instructional Note: Rigid motions are at the foundation of the definition of congruence. Students reason from the basic properties of rigid motions (that they preserve distance and

	angle), which are assumed without proof. Rigid motions and their assumed properties can be used to establish the usual triangle congruence criteria, which can then be used to prove other theorems.
M.1HS8.55	Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent. Instructional Note: Rigid motions are at the foundation of the definition of congruence. Students reason from the basic properties of rigid motions (that they preserve distance and angle), which are assumed without proof. Rigid motions and their assumed properties can be used to establish the usual triangle congruence criteria, which can then be used to prove other theorems.
M.1HS8.56	Explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow from the definition of congruence in terms of rigid motions. Instructional Note: Rigid motions are at the foundation of the definition of congruence. Students reason from the basic properties of rigid motions (that they preserve distance and angle), which are assumed without proof. Rigid motions and their assumed properties can be used to establish the usual triangle congruence criteria, which can then be used to prove other theorems.

Cluster	**Make geometric constructions.**
M.1HS8.57	Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.). Copying a segment; copying an angle; bisecting a segment; bisecting an angle; constructing perpendicular lines, including the perpendicular bisector of a line segment; and constructing a line parallel to a given line through a point not on the line. Instructional Note: Build on prior student experience with simple constructions. Emphasize the ability to formalize and defend how these constructions result in the desired objects. Some of these constructions are closely related to previous standards and can be introduced in conjunction with them.
M.1HS8.58	Construct an equilateral triangle, a square and a regular hexagon inscribed in a circle. Instructional Note: Build on prior student experience with simple constructions. Emphasize the ability to formalize and defend how these constructions result in the desired objects. Some of these constructions are closely related to previous standards and can be introduced in conjunction with them.

Cluster	**Understand and apply the Pythagorean theorem.**
M.1HS8.59	Explain a proof of the Pythagorean theorem and its converse.
M.1HS8.60	Apply the Pythagorean theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions. Instructional Note: Discuss applications of the Pythagorean theorem and its connections to radicals, rational exponents, and irrational numbers.
M.1HS8.61	Apply the Pythagorean theorem to find the distance between two points in a coordinate system. Instructional Note: Discuss applications of the Pythagorean theorem and its connections to radicals, rational exponents, and irrational numbers.

Connecting Algebra and Geometry through Coordinates

Cluster	**Use coordinates to prove simple geometric theorems algebraically.**

M.1HS8.62	Use coordinates to prove simple geometric theorems algebraically. (e.g., Prove or disprove that a figure defined by four given points in the coordinate plane is a rectangle; prove or disprove that the point (1, √3) lies on the circle centered at the origin and containing the point (0, 2).) Instructional Note: Reasoning with triangles in this unit is limited to right triangles (e.g., derive the equation for a line through two points using similar right triangles).
M.1HS8.63	Prove the slope criteria for parallel and perpendicular lines; use them to solve geometric problems. (e.g., Find the equation of a line parallel or perpendicular to a given line that passes through a given point.) Instructional Note: Reasoning with triangles in this unit is limited to right triangles (e.g., derive the equation for a line through two points using similar right triangles). Relate work on parallel lines to work on M.1HS8.35 involving systems of equations having no solution or infinitely many solutions.
M.1HS8.64	Use coordinates to compute perimeters of polygons and areas of triangles and rectangles, (e.g., using the distance formula). Instructional Note: Reasoning with triangles in this unit is limited to right triangles (e.g., derive the equation for a line through two points using similar right triangles). This standard provides practice with the distance formula and its connection with the Pythagorean theorem.

Mathematics – High School Mathematics I

All West Virginia teachers are responsible for classroom instruction that integrates content standards and objectives and mathematical habits of mind. Students in this course will focus on six critical units that deepen and extend understanding of linear relationships, in part by contrasting them with exponential phenomena, and in part by applying linear models to data that exhibit a linear trend. Students in Mathematics 1 will use properties and theorems involving congruent figures to deepen and extend understanding of geometric knowledge from prior grades and develop connections between the algebraic and geometric ideas studied. Mathematical habits of mind, which should be integrated in these content areas, include: making sense of problems and persevering in solving them, reasoning abstractly and quantitatively; constructing viable arguments and critiquing the reasoning of others; modeling with mathematics; using appropriate tools strategically; attending to precision, looking for and making use of structure; and looking for and expressing regularity in repeated reasoning. Students will continue developing mathematical proficiency in a developmentally-appropriate progressions of standards. Continuing the skill progressions from previous courses, the following chart represents the mathematical understandings that will be developed:

Relationships between Quantities	Linear and Exponential Relationships
• Solve problems with a wide range of units and solve problems by thinking about units. (e.g., “The Trans Alaska Pipeline System is 800 miles long and cost $8 billion to build. Divide one of these numbers by the other. What is the meaning of the answer?”; “Greenland has a population of 56,700 and a land area of 2,175,600 square kilometers. By what factor is the population density of the United States, 80 persons per square mile, larger than the population density of Greenland?”)	• Understand contextual relationships of variables and constants. (e.g., Annie is picking apples with her sister. The number of apples in her basket is described by n = 22t + 12, where t is the number of minutes Annie spends picking apples. What do the numbers 22 and 12 tell you about Annie’s apple picking?)
Reasoning with Equations	**Descriptive Statistics**
• Translate between various forms of linear equations. (e.g., The perimeter of a rectangle is given by P = 2W + 2L. Solve for W and restate in words the meaning of this new formula in terms of the meaning of the other variables.) • Explore systems of equations, find and interpret their solutions. (e.g., The high school is putting on the musical Footloose. The auditorium has 300 seats. Student tickets are $3 and adult tickets are $5. The royalty for the musical is $1300. What combination of student and adult tickets do you need to fill the house and pay the royalty? How could you change the price of tickets so more students can go?)	• Use linear regression techniques to describe the relationship between quantities and assess the fit of the model. (e.g., Use the high school and university grades for 250 students to create a model that can be used to predict a student’s university GPA based on his high school GPA.)

Congruence, Proof, and Constructions	Connecting Algebra and Geometry through Coordinates
• Given a transformation, work backwards to discover the sequence that led to the transformation. • Given two quadrilaterals that are reflections of each other, find the line of that reflection.	• Use a rectangular coordinate system and build on understanding of the Pythagorean Theorem to find distances. (e.g., Find the area and perimeter of a real-world shape using a coordinate grid and Google Earth.) • Analyze the triangles and quadrilaterals on the coordinate plane to determine their properties. (e.g., Determine whether a given quadrilateral is a rectangle.)

Numbering of Standards

The following Mathematics Standards will be numbered continuously. The following ranges relate to the clusters found within Mathematics:

Relationships between Quantities	
Reason quantitatively and use units to solve problems.	Standards 1-3
Interpret the structure of expressions.	Standard 4
Create equations that describe numbers or relationships.	Standards 5-8
Linear and Exponential Relationships	
Represent and solve equations and inequalities graphically.	Standards 9-11
Understand the concept of a function and use function notation.	Standards 12-14
Interpret functions that arise in applications in terms of a context.	Standards 15-17
Analyze functions using different representations.	Standards 18-19
Build a function that models a relationship between two quantities.	Standards 20-21
Build new functions from existing functions.	Standards 22
Construct and compare linear, quadratic, and exponential models and solve problems.	Standards 23-25
Interpret expressions for functions in terms of the situation they model.	Standard 26
Reasoning with Equations	
Understand solving equations as a process of reasoning and explain the reasoning.	Standard 27
Solve equations and inequalities in one variable.	Standard 28
Solve systems of equations.	Standards 29-30
Descriptive Statistics	
Summarize, represent, and interpret data on a single count or measurement variable.	Standards 31-33
Summarize, represent, and interpret data on two	Standards 34-35

categorical and quantitative variables.	
Interpret linear models.	Standards 36-38
Congruence, Proof, and Constructions	
Experiment with transformations in the plane.	Standards 39-43
Understand congruence in terms of rigid motions.	Standards 44-46
Make geometric constructions.	Standards 47-48
Connecting Algebra and Geometry through Coordinates	
Use coordinates to prove simple geometric theorems algebraically.	Standards 49-51

Relationships between Quantities

Cluster	Reason quantitatively and use units to solve problems.
M.1HS.1	Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.
M.1HS.2	Define appropriate quantities for the purpose of descriptive modeling. Instructional Note: Working with quantities and the relationships between them provides grounding for work with expressions, equations, and functions.
M.1HS.3	Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.

Cluster	Interpret the structure of expressions.
M.1HS.4	Interpret expressions that represent a quantity in terms of its context.* a. Interpret parts of an expression, such as terms, factors, and coefficients. b. Interpret complicated expressions by viewing one or more of their parts as a single entity. For example, interpret $P(1 + r)^n$ as the product of P and a factor not depending on P. Instructional Note: Limit to linear expressions and to exponential expressions with integer exponents.

Cluster	Create equations that describe numbers or relationships.
M.1HS.5	Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions and simple rational and exponential functions. Instructional Note: Limit to linear and exponential equations and in the case of exponential equations, limit to situations requiring evaluation of exponential functions at integer inputs.
M.1HS.6	Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales. Instructional Note: Limit to linear and exponential equations and in the case of exponential equations, limit to situations requiring evaluation of exponential functions at integer inputs.
M.1HS.7	Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context. (e.g., Represent inequalities describing nutritional and cost constraints on combinations of different foods.) Instructional Note: Limit to linear equations and inequalities.

M.1HS.8	Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. (e.g., Rearrange Ohm's law V = IR to highlight resistance R. Instructional Note: Limit to formulas with a linear focus.

Linear and Exponential Relationships

Cluster	**Represent and solve equations and inequalities graphically.**
M.1HS.9	Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line). Instructional Note: Focus on linear and exponential equations and be able to adapt and apply that learning to other types of equations in future courses.
M.1HS.10	Explain why the x-coordinates of the points where the graphs of the equations y = f(x) and y = g(x) intersect are the solutions of the equation f(x) = g(x); find the solutions approximately, (e.g., using technology to graph the functions, make tables of values, or find successive approximations). Include cases where f(x) and/or g(x) are linear, polynomial, rational, absolute value exponential, and logarithmic functions. Instructional Note: Focus on cases where f(x) and g(x) are linear or exponential.
M.1HS.11	Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality) and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.

Cluster	**Understand the concept of a function and use function notation.**
M.1HS.12	Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then f(x) denotes the output of f corresponding to the input x. The graph of f is the graph of the equation y = f(x). Instructional Note: Students should experience a variety of types of situations modeled by functions. Detailed analysis of any particular class of function at this stage is not advised. Students should apply these concepts throughout their future mathematics courses. Draw examples from linear and exponential functions.
M.1HS.13	Use function notation, evaluate functions for inputs in their domains and interpret statements that use function notation in terms of a context. Instructional Note: Students should experience a variety of types of situations modeled by functions. Detailed analysis of any particular class of function at this stage is not advised. Students should apply these concepts throughout their future mathematics courses. Draw examples from linear and exponential functions.
M.1HS.14	Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. For example, the Fibonacci sequence is defined recursively by f(0) = f(1) = 1, f(n+1) = f(n)+ f(n-1) for n ≥ 1. Instructional Note: Students should experience a variety of types of situations modeled by functions. Detailed analysis of any particular class of function at this stage is not advised. Students should apply these concepts throughout their future mathematics courses. Draw examples from linear and exponential functions. Draw connection to M.1HS.21, which requires students to write arithmetic and geometric sequences. Emphasize arithmetic and geometric sequences as examples of linear and exponential functions.

Cluster	Interpret functions that arise in applications in terms of a context.
M.1HS.15	For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive or negative; relative maximums and minimums; symmetries; end behavior; and periodicity. Instructional Note: Focus on linear and exponential functions.
M.1HS.16	Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. (e.g., If the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.) Instructional Note: Focus on linear and exponential functions.
M.1HS.17	Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph. Instructional Note: Focus on linear functions and intervals for exponential functions whose domain is a subset of the integers. Mathematics II and III will address other function types.

Cluster	Analyze functions using different representations.
M.1HS.18	Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. a. Graph linear and quadratic functions and show intercepts, maxima, and minima. b. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude. Instructional Note: Focus on linear and exponential functions. Include comparisons of two functions presented algebraically. For example, compare the growth of two linear functions, or two exponential functions such as $y = 3^n$ and $y = 100 \cdot 2^n$.
M.1HS.19	Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). (e.g., Given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.) Instructional Note: Focus on linear and exponential functions. Include comparisons of two functions presented algebraically. For example, compare the growth of two linear functions, or two exponential functions such as $y = 3^n$ and $y = 100 \cdot 2^n$.

Cluster	Build a function that models a relationship between two quantities.
M.1HS.20	Write a function that describes a relationship between two quantities. a. Determine an explicit expression, a recursive process or steps for calculation from a context. b. Combine standard function types using arithmetic operations. (e.g., Build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.) Instructional Note: Limit to linear and exponential functions.
M.1HS.21	Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.

	Instructional Note: Limit to linear and exponential functions. Connect arithmetic sequences to linear functions and geometric sequences to exponential functions.

Cluster	**Build new functions from existing functions.**
M.1HS.22	Identify the effect on the graph of replacing f(x) by f(x) + k, k f(x), f(kx), and f(x + k) for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them. Instructional Note: Focus on vertical translations of graphs of linear and exponential functions. Relate the vertical translation of a linear function to its y-intercept. While applying other transformations to a linear graph is appropriate at this level, it may be difficult for students to identify or distinguish between the effects of the other transformations included in this standard.

Cluster	**Construct and compare linear, quadratic, and exponential models and solve problems.**
M.1HS.23	Distinguish between situations that can be modeled with linear functions and with exponential functions. a. Prove that linear functions grow by equal differences over equal intervals; exponential functions grow by equal factors over equal intervals. b. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another. c. Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.
M.1HS.24	Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).
M.1HS.25	Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function. Instructional Note: Limit to comparisons between exponential and linear models.

Cluster	**Interpret expressions for functions in terms of the situation they model.**
M.1HS.26	Interpret the parameters in a linear or exponential function in terms of a context. Instructional Note: Limit exponential functions to those of the form $f(x) = b^x + k$.

Reasoning with Equations

Cluster	**Understand solving equations as a process of reasoning and explain the reasoning.**
M.1HS.27	Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method. Instructional Note: Students should focus on linear equations and be able to extend and apply their reasoning to other types of equations in future courses. Students will solve exponential equations with logarithms in Mathematics III.

Cluster	**Solve equations and inequalities in one variable.**

M.1HS.28	Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters. Instructional Note: Extend earlier work with solving linear equations to solving linear inequalities in one variable and to solving literal equations that are linear in the variable being solved for. Include simple exponential equations that rely only on application of the laws of exponents, such as $5^x = 125$ or $2^x = 1/16$.

Cluster	**Solve systems of equations.**
M.1HS.29	Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions. Instructional Note: Build on student experiences graphing and solving systems of linear equations from middle school to focus on justification of the methods used. Include cases where the two equations describe the same line (yielding infinitely many solutions) and cases where two equations describe parallel lines (yielding no solution); connect to M.1HS.50, which requires students to prove the slope criteria for parallel lines.
M.1HS.30	Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables. Instructional Note: Build on student experiences graphing and solving systems of linear equations from middle school to focus on justification of the methods used. Include cases where the two equations describe the same line (yielding infinitely many solutions) and cases where two equations describe parallel lines (yielding no solution); connect to M.1HS.50, which requires students to prove the slope criteria for parallel lines.

Descriptive Statistics

Cluster	**Summarize, represent, and interpret data on a single count or measurement variable.**
M.1HS.31	Represent data with plots on the real number line (dot plots, histograms, and box plots).
M.1HS.32	Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets. Instructional Note: In grades 6 – 8, students describe center and spread in a data distribution. Here they choose a summary statistic appropriate to the characteristics of the data distribution, such as the shape of the distribution or the existence of extreme data points.
M.1HS.33	Interpret differences in shape, center and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers). Instructional Note: In grades 6 – 8, students describe center and spread in a data distribution. Here they choose a summary statistic appropriate to the characteristics of the data distribution, such as the shape of the distribution or the existence of extreme data points.

Cluster	**Summarize, represent, and interpret data on two categorical and quantitative variables.**
M.1HS.34	Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal and conditional relative frequencies). Recognize possible associations and trends in the

	data.
M.1HS.35	Represent data on two quantitative variables on a scatter plot, and describe how the variables are related. a. Fit a function to the data; use functions fitted to data to solve problems in the context of the data. Use given functions or choose a function suggested by the context. Emphasize linear and exponential models. b. Informally assess the fit of a function by plotting and analyzing residuals. (Focus should be on situations for which linear models are appropriate.) c. Fit a linear function for scatter plots that suggest a linear association. Instructional Note: Students take a more sophisticated look at using a linear function to model the relationship between two numerical variables. In addition to fitting a line to data, students assess how well the model fits by analyzing residuals.

Cluster	**Interpret linear models.**
M.1HS.36	Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data. Instructional Note: Build on students' work with linear relationships in eighth grade and introduce the correlation coefficient. The focus here is on the computation and interpretation of the correlation coefficient as a measure of how well the data fit the relationship.
M.1HS.37	Compute (using technology) and interpret the correlation coefficient of a linear fit. Instructional Note: Build on students' work with linear relationships in eighth grade and introduce the correlation coefficient. The focus here is on the computation and interpretation of the correlation coefficient as a measure of how well the data fit the relationship.
M.1HS.38	Distinguish between correlation and causation. Instructional Note: The important distinction between a statistical relationship and a cause-and-effect relationship arises here.

Congruence, Proof, and Constructions

Cluster	**Experiment with transformations in the plane.**
M.1HS.39	Know precise definitions of angle, circle, perpendicular line, parallel line and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.
M.1HS.40	Represent transformations in the plane using, for example, transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not (e.g., translation versus horizontal stretch). Instructional Note: Build on student experience with rigid motions from earlier grades. Point out the basis of rigid motions in geometric concepts, (e.g., translations move points a specified distance along a line parallel to a specified line; rotations move objects along a circular arc with a specified center through a specified angle).
M.1HS.41	Given a rectangle, parallelogram, trapezoid or regular polygon, describe the rotations and reflections that carry it onto itself. Instructional Note: Build on student experience with rigid motions from earlier grades. Point out the basis of rigid motions in geometric concepts, (e.g., translations move points a specified distance along a line

	parallel to a specified line; rotations move objects along a circular arc with a specified center through a specified angle).
M.1HS.42	Develop definitions of rotations, reflections and translations in terms of angles, circles, perpendicular lines, parallel lines and line segments. Instructional Note: Build on student experience with rigid motions from earlier grades. Point out the basis of rigid motions in geometric concepts, (e.g., translations move points a specified distance along a line parallel to a specified line; rotations move objects along a circular arc with a specified center through a specified angle).
M.1HS.43	Given a geometric figure and a rotation, reflection or translation draw the transformed figure using, e.g., graph paper, tracing paper or geometry software. Specify a sequence of transformations that will carry a given figure onto another. Instructional Note: Build on student experience with rigid motions from earlier grades. Point out the basis of rigid motions in geometric concepts, (e.g., translations move points a specified distance along a line parallel to a specified line; rotations move objects along a circular arc with a specified center through a specified angle).

Cluster	**Understand congruence in terms of rigid motions.**
M.1HS.44	Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent. Instructional Note: Rigid motions are at the foundation of the definition of congruence. Students reason from the basic properties of rigid motions (that they preserve distance and angle), which are assumed without proof. Rigid motions and their assumed properties can be used to establish the usual triangle congruence criteria, which can then be used to prove other theorems.
M.1HS.45	Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent. Instructional Note: Rigid motions are at the foundation of the definition of congruence. Students reason from the basic properties of rigid motions (that they preserve distance and angle), which are assumed without proof. Rigid motions and their assumed properties can be used to establish the usual triangle congruence criteria, which can then be used to prove other theorems.
M.1HS.46	Explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow from the definition of congruence in terms of rigid motions. Instructional Note: Rigid motions are at the foundation of the definition of congruence. Students reason from the basic properties of rigid motions (that they preserve distance and angle), which are assumed without proof. Rigid motions and their assumed properties can be used to establish the usual triangle congruence criteria, which can then be used to prove other theorems.

Cluster	**Make geometric constructions.**
M.1HS.47	Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.). Copying a segment; copying an angle; bisecting a segment; bisecting an angle; constructing perpendicular lines, including the perpendicular bisector of a line segment; and constructing a line parallel to a given line through a point not on the line. Instructional Note: Build on prior student experience with simple

	constructions. Emphasize the ability to formalize and defend how these constructions result in the desired objects. Some of these constructions are closely related to previous standards and can be introduced in conjunction with them.
M.1HS.48	Construct an equilateral triangle, a square and a regular hexagon inscribed in a circle. Instructional Note: Build on prior student experience with simple constructions. Emphasize the ability to formalize and defend how these constructions result in the desired objects. Some of these constructions are closely related to previous standards and can be introduced in conjunction with them.

Connecting Algebra and Geometry through Coordinates

Cluster	**Use coordinates to prove simple geometric theorems algebraically.**
M.1HS.49	Use coordinates to prove simple geometric theorems algebraically. (e.g., Prove or disprove that a figure defined by four given points in the coordinate plane is a rectangle; prove or disprove that the point (1, √3) lies on the circle centered at the origin and containing the point (0, 2).) Instructional Note: Reasoning with triangles in this unit is limited to right triangles (e.g., derive the equation for a line through two points using similar right triangles).
M.1HS.50	Prove the slope criteria for parallel and perpendicular lines; use them to solve geometric problems. (e.g., Find the equation of a line parallel or perpendicular to a given line that passes through a given point.) Instructional Note: Reasoning with triangles in this unit is limited to right triangles (e.g., derive the equation for a line through two points using similar right triangles). Relate work on parallel lines to work on M.1HS.29 involving systems of equations having no solution or infinitely many solutions.
M.1HS.51	Use coordinates to compute perimeters of polygons and areas of triangles and rectangles, (e.g., using the distance formula). Instructional Note: Reasoning with triangles in this unit is limited to right triangles (e.g., derive the equation for a line through two points using similar right triangles). This standard provides practice with the distance formula and its connection with the Pythagorean theorem.

Mathematics – High School Mathematics II

All West Virginia teachers are responsible for classroom instruction that integrates content standards and objectives and mathematical habits of mind. Students in this course will focus on the need to extend the set of rational numbers, introducing real and complex numbers so that all quadratic equations can be solved. Students will explore the link between probability and data through conditional probability and counting methods, including their use in making and evaluating decisions. The study of similarity will lead students in Mathematics II to an understanding of right triangle trigonometry and connections to quadratics through Pythagorean relationships. Students will explore circles, with their quadratic algebraic representations. Mathematical habits of mind, which should be integrated in these content areas, include: making sense of problems and persevering in solving them, reasoning abstractly and quantitatively; constructing viable arguments and critiquing the reasoning of others; modeling with mathematics; using appropriate tools strategically; attending to precision, looking for and making use of structure; and looking for and expressing regularity in repeated reasoning. Students will continue developing mathematical proficiency in a developmentally-appropriate progressions of standards. Continuing the skill progressions from previous courses, the following chart represents the mathematical understandings that will be developed:

Extending the Number System	Quadratic Functions and Modeling
• Apply and reinforce laws of exponents to convert between radical notation and rational exponent notation; extend the properties of integer exponents to rational exponents and use them to simplify expressions. (e.g., $\sqrt[3]{16} = \sqrt[3]{2^4} = 2^{4/3}$; $((2^{-4})(2^{-4})^{\frac{1}{4}} = 2^{-1} = \frac{1}{2}$.)	• Find an explicit algebraic expression or series of steps to model the context with mathematical representations. (e.g., The total revenue for a company is found by multiplying the price per unit by the number of units sold minus the production cost. The price per unit is modeled by p(n) = -0.5n^2 + 6. The number of units sold is n. Production cost is modeled by c(n) = 3n + 7. Write the revenue function.)
Expressions and Equations	**Applications of Probability**
• Solve a system consisting of a linear equation and a quadratic equation in two variables. (e.g., Find the intersection of the circle with a radius of 1 centered at the origin and the line y = -3(x - 2). Show your work both graphically and algebraically.)	• Work with probability and using ideas from probability in everyday situations. (e.g., Compare the chance that a person who smokes will develop lung cancer to the chance that a person who develops lung cancer smokes.)
Similarity, Right Triangle Trigonometry, and Proof	**Circles With and Without Coordinates**
• Apply knowledge of trigonometric ratios and the Pythagorean Theorem to determine distances in realistic situations. (e.g., Determine heights of inaccessible objects using various instruments, such as clinometers, hypsometers, transits, etc.)	• Use coordinates and equations to describe geometric properties algebraically. (e.g., Write the equation for a circle in the plane with specified center and radius.)

Numbering of Standards

The following Mathematics Standards will be numbered continuously. The following ranges relate to the clusters found within Mathematics:

Extending the Number System	
Extend the properties of exponents to rational exponents.	Standards 1-2
Use properties of rational and irrational numbers.	Standard 3
Perform arithmetic operations with complex numbers.	Standards 4-5
Perform arithmetic operations on polynomials.	Standard 6
Quadratic Functions and Modeling	
Interpret functions that arise in applications in terms of a context.	Standards 7-9
Analyze functions using different representations.	Standards 10-12
Build a function that models a relationship between two quantities.	Standard 13
Build new functions from existing functions.	Standards 14-15
Construct and compare linear, quadratic, and exponential models and solve problems.	Standard 16
Expressions and Equations	
Interpret the structure of expressions.	Standards 17-18
Write expressions in equivalent forms to solve problems.	Standard 19
Create equations that describe numbers or relationships.	Standards 20-22
Solve equations and inequalities in one variable.	Standard 23
Use complex numbers in polynomial identities and equations.	Standards 24-26
Solve systems of equations.	Standard 27
Applications of Probability	
Understand independence and conditional probability and use them to interpret data.	Standards 28-32
Use the rules of probability to compute probabilities of compound events in a uniform probability model.	Standards 33-36
Use probability to evaluate outcomes of decisions.	Standards 37-38
Similarity, Right Triangle Trigonometry, and Proof	
Understand similarity in terms of similarity transformations.	Standards 39-41
Prove geometric theorems.	Standards 42-44
Prove theorems involving similarity.	Standards 45-46
Use coordinates to prove simple geometric theorems algebraically.	Standard 47
Define trigonometric ratios and solve problems involving right triangles.	Standards 48-50
Prove and apply trigonometric identities.	Standard 51

Circles With and Without Coordinates	
Understand and apply theorems about circles.	Standards 52-55
Find arc lengths and areas of sectors of circles.	Standard 56
Translate between the geometric description and the equation for a conic section.	Standards 57-58
Use coordinates to prove simple geometric theorems algebraically.	Standard 59
Explain volume formulas and use them to solve problems.	Standards 60-61

Relationships between Quantities

Cluster	Extend the properties of exponents to rational exponents.
M.2HS.1	Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents. (e.g., We define $5^{1/3}$ to be the cube root of 5 because we want $(5^{1/3})^3 = 5^{(1/3)3}$ to hold, so $(5^{1/3})^3$ must equal 5.)
M.2HS.2	Rewrite expressions involving radicals and rational exponents using the properties of exponents.

Cluster	Use properties of rational and irrational numbers.
M.2HS.3	Explain why sums and products of rational numbers are rational, that the sum of a rational number and an irrational number is irrational and that the product of a nonzero rational number and an irrational number is irrational. Instructional Note: Connect to physical situations, e.g., finding the perimeter of a square of area 2.

Cluster	Perform arithmetic operations with complex numbers.
M.2HS.4	Know there is a complex number i such that $i^2 = -1$, and every complex number has the form a + bi with a and b real.
M.2HS.5	Use the relation $i^2 = -1$ and the commutative, associative and distributive properties to add, subtract and multiply complex numbers. Instructional Note: Limit to multiplications that involve i^2 as the highest power of i

Cluster	Perform arithmetic operations on polynomials.
M.2HS.6	Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract and multiply polynomials. Instructional Note: Focus on polynomial expressions that simplify to forms that are linear or quadratic in a positive integer power of x.

Quadratic Functions and Modeling

Cluster	Interpret functions that arise in applications in terms of a context.
M.2HS.7	For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive or negative;

	relative maximums and minimums; symmetries; end behavior; and periodicity. Instructional Note: Focus on quadratic functions; compare with linear and exponential functions studied in Mathematics I.
M.2HS.8	Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. (e.g., If the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.) Instructional Note: Focus on quadratic functions; compare with linear and exponential functions studied in Mathematics I.
M.2HS.9	Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph. Instructional Note: Focus on quadratic functions; compare with linear and exponential functions studied in Mathematics I.

Cluster	**Analyze functions using different representations.**
M.2HS.10	Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. a. Graph linear and quadratic functions and show intercepts, maxima, and minima. b. Graph square root, cube root and piecewise-defined functions, including step functions and absolute value functions. Instructional Note: Compare and contrast absolute value, step and piecewise defined functions with linear, quadratic, and exponential functions. Highlight issues of domain, range and usefulness when examining piecewise-defined functions. Instructional Note: Extend work with quadratics to include the relationship between coefficients and roots and that once roots are known, a quadratic equation can be factored.
M.2HS.11	Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. a. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values and symmetry of the graph and interpret these in terms of a context. b. Use the properties of exponents to interpret expressions for exponential functions. (e.g., Identify percent rate of change in functions such as $y = (1.02)^{t,}$ $y = (0.97)^t$, $y = (1.01)^{12t}$, $y = (1.2)^{t/10}$, and classify them as representing exponential growth or decay.) Instructional Note: This unit and, in particular, this standard extends the work begun in Mathematics I on exponential functions with integer exponents. Instructional Note: Extend work with quadratics to include the relationship between coefficients and roots and that once roots are known, a quadratic equation can be factored.
M.2HS.12	Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). (e.g., Given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum). Instructional Note: Focus on expanding the types of functions considered to include, linear, exponential and quadratic. Extend work with quadratics to include the relationship between coefficients and roots and that once roots are known, a quadratic equation can be factored.

Cluster	Build a function that models a relationship between two quantities.
M.2HS.13	Write a function that describes a relationship between two quantities. a. Determine an explicit expression, a recursive process or steps for calculation from a context. b. Combine standard function types using arithmetic operations. (e.g., Build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model. Instructional Note: Focus on situations that exhibit a quadratic or exponential relationship.

Cluster	Build new functions from existing functions.
M.2HS.14	Identify the effect on the graph of replacing f(x) by f(x) + k, k f(x), f(kx), and f(x + k) for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them. Instructional Note: Focus on quadratic functions and consider including absolute value functions.
M.2HS.15	Find inverse functions. Solve an equation of the form f(x) = c for a simple function f that has an inverse and write an expression for the inverse. For example, $f(x)= 2 x^3$ or f(x) = (x+1)/(x-1) for x ≠ 1. Instructional Note: Focus on linear functions but consider simple situations where the domain of the function must be restricted in order for the inverse to exist, such as $f(x) = x^2$, x > 0.

Cluster	Construct and compare linear, quadratic, and exponential models and solve problems.
M.2HS.16	Using graphs and tables, observe that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically; or (more generally) as a polynomial function. Instructional Note: Compare linear and exponential growth studied in Mathematics I to quadratic growth.

Expressions and Equations

Cluster	Interpret the structure of expressions.
M.2HS.17	Interpret expressions that represent a quantity in terms of its context. a. Interpret parts of an expression, such as terms, factors, and coefficients. b. Interpret complicated expressions by viewing one or more of their parts as a single entity. For example, interpret $P(1 + r)^n$ as the product of P and a factor not depending on P. Instructional Note: Focus on quadratic and exponential expressions. Exponents are extended from the integer exponents found in Mathematics I to rational exponents focusing on those that represent square or cube roots.
M.2HS.18	Use the structure of an expression to identify ways to rewrite it. For example, see $x^4 – y^4$ as $(x^2)^2 – (y^2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2 – y^2)(x^2 + y^2)$. Instructional Note: Focus on quadratic and exponential expressions.

Cluster	Write expressions in equivalent forms to solve problems.

M.2HS.19	Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. a. Factor a quadratic expression to reveal the zeros of the function it defines. b. Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines. c. Use the properties of exponents to transform expressions for exponential functions. For example the expression 1.15^t can be rewritten as $(1.15^{1/12})^{12t} \approx 1.012^{12t}$ to reveal the approximate equivalent monthly interest rate if the annual rate is 15%. Instructional Note: It is important to balance conceptual understanding and procedural fluency in work with equivalent expressions. For example, development of skill in factoring and completing the square goes hand-in-hand with understanding what different forms of a quadratic expression reveal.

Cluster	**Create equations that describe numbers or relationships.**
M.2HS.20	Create equations and inequalities in one variable and use them to solve problems. Instructional Note: Include equations arising from linear and quadratic functions, and simple rational and exponential functions. Extend work on linear and exponential equations in Mathematics I to quadratic equations.
M.2HS.21	Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales. Instructional Note: Extend work on linear and exponential equations in Mathematics I to quadratic equations.
M.2HS.22	Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. (e.g., Rearrange Ohm's law V = IR to highlight resistance R.) Instructional Note: Extend to formulas involving squared variables. Extend work on linear and exponential equations in Mathematics I to quadratic equations.

Cluster	**Solve equations and inequalities in one variable.**
M.2HS.23	Solve quadratic equations in one variable. a. Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x – p)^2 = q$ that has the same solutions. Derive the quadratic formula from this form. b. Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as a ± bi for real numbers a and b. Instructional Note: Extend to solving any quadratic equation with real coefficients, including those with complex solutions.

Cluster	**Use complex numbers in polynomial identities and equations.**
M.2HS.24	Solve quadratic equations with real coefficients that have complex solutions. Instructional Note: Limit to quadratics with real coefficients.
M.2HS.25(+)	Extend polynomial identities to the complex numbers. For example, rewrite $x^2 + 4$ as $(x + 2i)(x – 2i)$. Instructional Note: Limit to quadratics with real coefficients.
M.2HS.26(+)	Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials. Instructional Note: Limit to quadratics with real coefficients.

Cluster	**Solve systems of equations**.
M.2HS.27	Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically. (e.g., Find the points of intersection between the line $y = -3x$ and the circle $x^2 + y^2 = 3$.) Instructional Note: Include systems that lead to work with fractions. (e.g., Finding the intersections between $x^2 + y^2 = 1$ and $y = (x+1)/2$ leads to the point (3/5, 4/5) on the unit circle, corresponding to the Pythagorean triple $3^2 + 4^2 = 5^2$.)

Applications of Probability

Cluster	**Understand independence and conditional probability and use them to interpret data.**
M.2HS.28	Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes or as unions, intersections or complements of other events (“or,” “and,” “not”).
M.2HS.29	Understand that two events A and B are independent if the probability of A and B occurring together is the product of their probabilities and use this characterization to determine if they are independent.
M.2HS.30	Understand the conditional probability of A given B as P(A and B)/P(B), and interpret independence of A and B as saying that the conditional probability of A given B is the same as the probability of A, and the conditional probability of B given A is the same as the probability of B.
M.2HS.31	Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities. (e.g., Collect data from a random sample of students in your school on their favorite subject among math, science and English. Estimate the probability that a randomly selected student from your school will favor science given that the student is in tenth grade. Do the same for other subjects and compare the results.) Instructional Note: Build on work with two-way tables from Mathematics I to develop understanding of conditional probability and independence.
M.2HS.32	Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations. (e.g., Compare the chance of having lung cancer if you are a smoker with the chance of being a smoker if you have lung cancer.)

Cluster	**Use the rules of probability to compute probabilities of compound events in a uniform probability model.**
M.2HS.33	Find the conditional probability of A given B as the fraction of B’s outcomes that also belong to A and interpret the answer in terms of the model.
M.2HS.34	Apply the Addition Rule, P(A or B) = P(A) + P(B) – P(A and B), and interpret the answer in terms of the model.
M.2HS.35(+)	Apply the general Multiplication Rule in a uniform probability model, P(A and B) = P(A)P(B|A) = P(B)P(A|B), and interpret the answer in terms of the model.
M.2HS.36(+)	Use permutations and combinations to compute probabilities of compound events and solve problems.

Cluster	Use probability to evaluate outcomes of decisions.
M.2HS.37(+)	Use probabilities to make fair decisions (e.g., drawing by lots or using a random number generator).
M.2HS.38(+)	Analyze decisions and strategies using probability concepts (e.g., product testing, medical testing, and/or pulling a hockey goalie at the end of a game). Instructional Note: This unit sets the stage for work in Mathematics III, where the ideas of statistical inference are introduced. Evaluating the risks associated with conclusions drawn from sample data (i.e., incomplete information) requires an understanding of probability concepts.

Similarity, Right Triangle Trigonometry, and Proof

Cluster	Understand similarity in terms of similarity transformations
M.2HS.39	Verify experimentally the properties of dilations given by a center and a scale factor. a. A dilation takes a line not passing through the center of the dilation to a parallel line and leaves a line passing through the center unchanged. b. The dilation of a line segment is longer or shorter in the ratio given by the scale factor.
M.2HS.40	Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides.
M.2HS.41	Use the properties of similarity transformations to establish the AA criterion for two triangles to be similar.

Cluster	Prove geometric theorems.
M.2HS.42	Prove theorems about lines and angles. Theorems include: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent; points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints. Implementation may be extended to include concurrence of perpendicular bisectors and angle bisectors as preparation for M.2HS.C.3. Instructional Note: Encourage multiple ways of writing proofs, such as in narrative paragraphs, using flow diagrams, in two-column format, and using diagrams without words. Students should be encouraged to focus on the validity of the underlying reasoning while exploring a variety of formats for expressing that reasoning.
M.2HS.43	Prove theorems about triangles. Theorems include: measures of interior angles of a triangle sum to 180°; base angles of isosceles triangles are congruent; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point. Instructional Note: Encourage multiple ways of writing proofs, such as in narrative paragraphs, using flow diagrams, in two-column format, and using diagrams without words. Students should be encouraged to focus on the validity of the underlying reasoning while exploring a variety of formats for expressing that reasoning. Implementation of this standard may be extended to include concurrence of perpendicular bisectors and angle bisectors in preparation for the unit on Circles With and Without Coordinates.
M.2HS.44	Prove theorems about parallelograms. Theorems include: opposite sides are

	congruent, opposite angles are congruent, the diagonals of a parallelogram bisect each other and conversely, rectangles are parallelograms with congruent diagonals. Instructional Note: Encourage multiple ways of writing proofs, such as in narrative paragraphs, using flow diagrams, in two-column format and using diagrams without words. Students should be encouraged to focus on the validity of the underlying reasoning while exploring a variety of formats for expressing that reasoning.

Cluster	**Prove theorems involving similarity.**
M.2HS.45	Prove theorems about triangles. Theorems include: a line parallel to one side of a triangle divides the other two proportionally and conversely; the Pythagorean Theorem proved using triangle similarity.
M.2HS.46	Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures.

Cluster	**Use coordinates to prove simple geometric theorems algebraically.**
M.2HS.47	Find the point on a directed line segment between two given points that partitions the segment in a given ratio.

Cluster	**Define trigonometric ratios and solve problems involving right triangles.**
M.2HS.48	Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles.
M.2HS.49	Explain and use the relationship between the sine and cosine of complementary angles.
M.2HS.50	Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.

Cluster	**Prove and apply trigonometric identities.**
M.2HS.51	Prove the Pythagorean identity $\sin^2(\theta) + \cos^2(\theta) = 1$ and use it to find sin(θ), cos (θ), or tan (θ), given sin (θ), cos (θ), or tan (θ), and the quadrant of the angle. Instructional Note: Limit θ to angles between 0 and 90 degrees. Connect with the Pythagorean theorem and the distance formula. Extension of trigonometric functions to other angles through the unit circle is included in Mathematics III.

Circles With and Without Coordinates

Cluster	**Understand and apply theorems about circles.**
M.2HS.52	Prove that all circles are similar.
M.2HS.53	Identify and describe relationships among inscribed angles, radii and chords. Include the relationship between central, inscribed and circumscribed angles; inscribed angles on a diameter are right angles; the radius of a circle is perpendicular to the tangent where the radius intersects the circle.
M.2HS.54	Construct the inscribed and circumscribed circles of a triangle and prove properties of angles for a quadrilateral inscribed in a circle.
M.2HS.55(+)	Construct a tangent line from a point outside a given circle to the circle.

Cluster	**Find arc lengths and areas of sectors of circles.**

M.2HS.56	Derive using similarity the fact that the length of the arc intercepted by an angle is proportional to the radius and define the radian measure of the angle as the constant of proportionality; derive the formula for the area of a sector. Instructional Note: Emphasize the similarity of all circles. Note that by similarity of sectors with the same central angle, arc lengths are proportional to the radius. Use this as a basis for introducing radian as a unit of measure. It is not intended that it be applied to the development of circular trigonometry in this course.

Cluster	**Translate between the geometric description and the equation for a conic section.**
M.2HS.57	Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation. Instructional Note: Connect the equations of circles and parabolas to prior work with quadratic equations.
M.2HS.58	Derive the equation of a parabola given the focus and directrix. Instructional Note: The directrix should be parallel to a coordinate axis.

Cluster	**Use coordinates to prove simple geometric theorems algebraically.**
M.2HS.59	Use coordinates to prove simple geometric theorems algebraically. (e.g., Prove or disprove that a figure defined by four given points in the coordinate plane is a rectangle; prove or disprove that the point (1, √3) lies on the circle centered at the origin and containing the point (0, 2).) Instructional Note: Include simple proofs involving circles.

Cluster	**Explain volume formulas and use them to solve problems.**
M.2HS.60	Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone. Use dissection arguments, Cavalieri's principle and informal limit arguments. Instructional Note: Informal arguments for area and volume formulas can make use of the way in which area and volume scale under similarity transformations: when one figure in the plane results from another by applying a similarity transformation with scale factor k, its area is k^2 times the area of the first.
M.2HS.61	Use volume formulas for cylinders, pyramids, cones and spheres to solve problems. Volumes of solid figures scale by k^3 under a similarity transformation with scale factor k.

Mathematics – High School Mathematics III (LA and STEM)
Math III LA course does not include the (+) standards.
Math III STEM course includes standards identified by (+) sign
Math III TR course (Technical Readiness) includes standards identified by (*)
Math IV TR course (Technical Readiness) includes standards identified by (^)

Math III Technical Readiness and Math IV Technical Readiness are course options (for juniors and seniors) built for the mathematics content of Math III through integration of career clusters. These courses integrate academics with hands-on career content. The collaborative teaching model is recommended based at our Career and Technical Education (CTE) centers. The involvement of a highly qualified Mathematics teacher and certified CTE teachers will ensure a rich, authentic and respectful environment for delivery of the academics in "real world" scenarios.

All West Virginia teachers are responsible for classroom instruction that integrates content standards and mathematical habits of mind. Students in this course will make connections and applications the accumulation of learning that they have from their previous courses, with content grouped into four critical units. Students will apply methods from probability and statistics to draw inferences and conclusions from data. They will expand their repertoire of functions to include polynomial, rational and radical functions and their study of right triangle trigonometry to include general triangles. Students will bring together their experiences with functions and geometry to create models and solve contextual problems. Mathematical habits of mind, which should be integrated in these content areas, include: making sense of problems and persevering in solving them, reasoning abstractly and quantitatively; constructing viable arguments and critiquing the reasoning of others; modeling with mathematics; using appropriate tools strategically; attending to precision, looking for and making use of structure; and looking for and expressing regularity in repeated reasoning. Students will continue developing mathematical proficiency in a developmentally-appropriate progressions of standards. Continuing the skill progressions from previous courses, the following chart represents the mathematical understandings that will be developed:

Inferences and Conclusions from Data	Polynomials, Rational, and Radical Relationships
• Make inferences and justify conclusions from sample surveys, experiments, and observational studies. Analyze decisions and strategies using probability concepts (e.g., product testing, medical testing, pulling a hockey goalie at the end of a game).	• Derive the formula for the sum of a geometric series, and use the formula to solve problems. (e.g., Calculate mortgage payments.)
Trigonometry of General Triangles and Trigonometric Functions	**Mathematical Modeling**
• Apply knowledge of the Law of Sines and the Law of Cosines to determine distances in realistic situations. (e.g., Determine heights of inaccessible objects.)	• Analyze real-world situations using mathematics to understand the situation better and optimize, troubleshoot, or make an informed decision. (e.g., Estimate water and food needs in a disaster area, or use volume formulas and graphs to find an optimal size for an industrial package.)

<u>Numbering of Standards</u>

The following Mathematics Standards will be numbered continuously. The following ranges relate to the clusters found within Mathematics:

Inferences and Conclusions from Data	
Summarize, represent, and interpret data on single count or measurement variable.	Standard 1
Understand and evaluate random processes underlying statistical experiments.	Standards 2-3
Make inferences and justify conclusions from sample surveys, experiments, and observational studies.	Standards 4-7
Use probability to evaluate outcomes of decisions.	Standards 8-9
Polynomials, Rational, and Radical Relationships	
Use complex numbers in polynomial identities and equations.	Standards 10-11
Interpret the structure of expressions.	Standards 12-13
Write expressions in equivalent forms to solve problems.	Standard 14
Perform arithmetic operations on polynomials.	Standard 15
Understand the relationship between zeros and factors of polynomials.	Standards 16-17
Use polynomial identities to solve problems.	Standards 18-19
Rewrite rational expressions.	Standards 20-21
Understand solving equations as a process of reasoning and explain the reasoning.	Standard 22
Represent and solve equations and inequalities graphically.	Standard 23
Analyze functions using different representations.	Standard 24
Trigonometry of General Triangles and Trigonometric Functions	
Apply trigonometry to general triangles.	Standards 25-27
Extend the domain of trigonometric functions using the unit circle.	Standards 28-29
Model periodic phenomena with trigonometric functions.	Standard 30
Mathematical Modeling	
Create equations that describe numbers or relationships.	Standards 31-34
Interpret functions that arise in applications in terms of a context.	Standards 35-37
Analyze functions using different representations.	Standards 38-40
Build a function that models a relationship between two quantities.	Standard 41
Build new functions from existing functions.	Standards 42-43
Construct and compare linear, quadratic, and exponential models and solve problems.	Standard 44

Visualize relationships between two dimensional and three-dimensional objects.	Standard 45
Apply geometric concepts in modeling situations.	Standards 46-48

Inferences and Conclusions from Data

Cluster	Summarize, represent, and interpret data on single count or measurement variable.
M.3HS.1(*)	Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets and tables to estimate areas under the normal curve. Instructional Note: While students may have heard of the normal distribution, it is unlikely that they will have prior experience using it to make specific estimates. Build on students' understanding of data distributions to help them see how the normal distribution uses area to make estimates of frequencies (which can be expressed as probabilities). Emphasize that only some data are well described by a normal distribution.

Cluster	Understand and evaluate random processes underlying statistical experiments.
M.3HS.2(*)	Understand that statistics allows inferences to be made about population parameters based on a random sample from that population.
M.3HS.3(*)	Decide if a specified model is consistent with results from a given data-generating process, for example, using simulation. (e.g., A model says a spinning coin falls heads up with probability 0.5. Would a result of 5 tails in a row cause you to question the model?) Instructional Note: Include comparing theoretical and empirical results to evaluate the effectiveness of a treatment.

Cluster	Make inferences and justify conclusions from sample surveys, experiments, and observational studies.
M.3HS.4 (*,^)	Recognize the purposes of and differences among sample surveys, experiments and observational studies; explain how randomization relates to each. Instructional Note: In earlier grades, students are introduced to different ways of collecting data and use graphical displays and summary statistics to make comparisons. These ideas are revisited with a focus on how the way in which data is collected determines the scope and nature of the conclusions that can be drawn from that data. The concept of statistical significance is developed informally through simulation as meaning a result that is unlikely to have occurred solely as a result of random selection in sampling or random assignment in an experiment.
M.3HS.5 (*,^)	Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling. Instructional Note: Focus on the variability of results from experiments—that is, focus on statistics as a way of dealing with, not eliminating, inherent randomness.
M.3HS.6 (*,^)	Use data from a randomized experiment to compare two treatments; use simulations to decide if differences between parameters are significant. Instructional Note: Focus on the variability of results from experiments—that is, focus on statistics as a way of dealing with, not eliminating, inherent randomness.

M.3HS.7 (*,^)	Evaluate reports based on data. Instructional Note: In earlier grades, students are introduced to different ways of collecting data and use graphical displays and summary statistics to make comparisons. These ideas are revisited with a focus on how the way in which data is collected determines the scope and nature of the conclusions that can be drawn from that data. The concept of statistical significance is developed informally through simulation as meaning a result that is unlikely to have occurred solely as a result of random selection in sampling or random assignment in an experiment.

Cluster	**Use probability to evaluate outcomes of decisions.**
M.3HS.8 (+, ^)	Use probabilities to make fair decisions (e.g., drawing by lots or using a random number generator).
M.3HS.9 (+, ^)	Analyze decisions and strategies using probability concepts (e.g., product testing, medical testing, and/or pulling a hockey goalie at the end of a game). Instructional Note: Extend to more complex probability models. Include situations such as those involving quality control or diagnostic tests that yields both false positive and false negative results.

Polynomials, Rational, and Radical Relationships

Cluster	**Use complex numbers in polynomial identities and equations.**
M.3HS.10 (+)	Extend polynomial identities to the complex numbers. For example, rewrite $x^2 + 4$ as $(x + 2i)(x – 2i)$. Instructional Note: Build on work with quadratics equations in Mathematics II. Limit to polynomials with real coefficients.
M.3HS.11 (+)	Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials.

Cluster	**Interpret the structure of expressions.**
M.3HS.12 (*)	Interpret expressions that represent a quantity in terms of its context. a. Interpret parts of an expression, such as terms, factors, and coefficients. b. Interpret complicated expressions by viewing one or more of their parts as a single entity. (e.g., Interpret $P(1 + r)^n$ as the product of P and a factor not depending on P.) Instructional Note: Extend to polynomial and rational expressions.
M.3HS.13 (*)	Use the structure of an expression to identify ways to rewrite it. For example, see $x^4 – y^4$ as $(x^2)^2 – (y^2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2 – y^2)(x^2 + y^2)$. Instructional Note: Extend to polynomial and rational expressions.

Cluster	**Write expressions in equivalent forms to solve problems.**
M.3HS.14 (^)	Derive the formula for the sum of a geometric series (when the common ratio is not 1), and use the formula to solve problems. (e.g., Calculate mortgage payments.) Instructional Note: Consider extending to infinite geometric series in curricular implementations of this course description.

Cluster	**Perform arithmetic operations on polynomials.**
M.3HS.15(*)	Understand that polynomials form a system analogous to the integers, namely, they

	are closed under the operations of addition, subtraction and multiplication; add, subtract and multiply polynomials. Instructional Note: Extend beyond the quadratic polynomials found in Mathematics II.

Cluster	Understand the relationship between zeros and factors of polynomials.
M.3HS.16 (*)	Know and apply the Remainder Theorem: For a polynomial p(x) and a number a, the remainder on division by x – a is p(a), so p(a) = 0 if and only if (x – a) is a factor of p(x).
M.3HS.17 (*)	Identify zeros of polynomials when suitable factorizations are available and use the zeros to construct a rough graph of the function defined by the polynomial.

Cluster	Use polynomial identities to solve problems.
M.3HS.18 (^)	Prove polynomial identities and use them to describe numerical relationships. For example, the polynomial identity $(x^2 + y^2)^2 = (x^2 – y^2)^2 + (2xy)^2$ can be used to generate Pythagorean triples. Instructional Note: This cluster has many possibilities for optional enrichment, such as relating the example in M.A2HS.10 to the solution of the system $u^2 + v^2 = 1$, $v = t(u+1)$, relating the Pascal triangle property of binomial coefficients to $(x + y)^{n+1} = (x + y)(x + y)^n$, deriving explicit formulas for the coefficients, or proving the binomial theorem by induction.
M.3HS.19 (+,^)	Know and apply the Binomial Theorem for the expansion of $(x + y)^n$ in powers of x and y for a positive integer n, where x and y are any numbers, with coefficients determined for example by Pascal's Triangle. Instructional Note: This cluster has many possibilities for optional enrichment, such as relating the example in M.A2HS.10 to the solution of the system $u^2 + v^2 = 1$, $v = t(u+1)$, relating the Pascal triangle property of binomial coefficients to $(x + y)^{n+1} = (x + y)(x + y)^n$, deriving explicit formulas for the coefficients, or proving the binomial theorem by induction.

Cluster	Rewrite rational expressions
M.3HS.20 (*)	Rewrite simple rational expressions in different forms; write a(x)/b(x) in the form q(x) + r(x)/b(x), where a(x), b(x), q(x), and r(x) are polynomials with the degree of r(x) less than the degree of b(x), using inspection, long division, or, for the more complicated examples, a computer algebra system. Instructional Note: The limitations on rational functions apply to the rational expressions.
M.3HS.21 (+)	Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply and divide rational expressions. Instructional Note: Requires the general division algorithm for polynomials.

Cluster	Understand solving equations as a process of reasoning and explain the reasoning.
M.3HS.22 (*)	Solve simple rational and radical equations in one variable and give examples showing how extraneous solutions may arise. Instructional Note: Extend to simple rational and radical equations.

Cluster	Represent and solve equations and inequalities graphically.
M.3HS.23 (*,^)	Explain why the x-coordinates of the points where the graphs of the equations y = f(x) and y = g(x) intersect are the solutions of the equation f(x) = g(x); find the

	solutions approximately (e.g., using technology to graph the functions, make tables of values or find successive approximations. Include cases where f(x) and/or g(x) are linear, polynomial, rational, absolute value, exponential and logarithmic functions. Instructional Note: Include combinations of linear, polynomial, rational, radical, absolute value, and exponential functions.

Cluster	**Analyze functions using different representations.**
M.3HS.24 (*,^)	Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. Graph polynomial functions, identifying zeros when suitable factorizations are available and showing end behavior. Instructional Note: Relate to the relationship between zeros of quadratic functions and their factored forms.

Trigonometry of General Triangles and Trigonometric Functions

Cluster	**Apply trigonometry to general triangles.**
M.3HS.25 (+,^)	Derive the formula A = 1/2 ab sin(C) for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side.
M.3HS.26 (+, ^)	Prove the Laws of Sines and Cosines and use them to solve problems. Instructional Note: With respect to the general case of the Laws of Sines and Cosines, the definitions of sine and cosine must be extended to obtuse angles.
M.3HS.27 (+,^)	Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles (e.g., surveying problems and/or resultant forces).

Cluster	**Extend the domain of trigonometric functions using the unit circle.**
M.3HS.28 (*)	Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.
M.3HS.29 (*)	Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.

Cluster	**Model periodic phenomena with trigonometric functions.**
M.3HS.30 (*)	Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.

Mathematical Modeling

Cluster	**Create equations that describe numbers or relationships.**
M.3HS.31 (*,^)	Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions. Instructional Note: Use all available types of functions to create such equations, including root functions, but constrain to simple cases.
M.3HS.32 (*,^)	Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales. Instructional Note: While functions will often be linear, exponential or quadratic the types of problems should draw from more complex situations than those addressed in

	Mathematics I. For example, finding the equation of a line through a given point perpendicular to another line allows one to find the distance from a point to a line.
M.3HS.33 (*,^)	Represent constraints by equations or inequalities and by systems of equations and/or inequalities and interpret solutions as viable or non-viable options in a modeling context. (e.g., Represent inequalities describing nutritional and cost constraints on combinations of different foods.)
M.3HS.34 (*,^)	Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. (e.g., Rearrange Ohm's law V = IR to highlight resistance R.) Instructional Note: The example given applies to earlier instances of this standard, not to the current course.

Cluster	**Interpret functions that arise in applications in terms of a context.**
M.3HS.35 (*)	For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive or negative; relative maximums and minimums; symmetries; end behavior; and periodicity. Instructional Note: Emphasize the selection of a model function based on behavior of data and context.
M.3HS.36 (*)	Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. (e.g., If the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.) Instructional Note: Emphasize the selection of a model function based on behavior of data and context.
M.3HS.37 (*)	Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph. Instructional Note: Emphasize the selection of a model function based on behavior of data and context.

Cluster	**Analyze functions using different representations.**
M.3HS.38 (*,^)	Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. a. Graph square root, cube root and piecewise-defined functions, including step functions and absolute value functions. b. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline and amplitude. Instructional Note: Focus on applications and how key features relate to characteristics of a situation, making selection of a particular type of function model appropriate.
M.3HS.39 (*,^)	Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. Instructional Note: Focus on applications and how key features relate to characteristics of a situation, making selection of a particular type of function model appropriate.
M.3HS.40 (*,^)	Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). (e.g., Given a graph of one quadratic function and an algebraic expression for another, say

	which has the larger maximum.) Instructional Note: Focus on applications and how key features relate to characteristics of a situation, making selection of a particular type of function model appropriate.

Cluster	**Build a function that models a relationship between two quantities.**
M.3HS.41 (*)	Write a function that describes a relationship between two quantities. Combine standard function types using arithmetic operations. (e.g., Build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.) Instructional Note: Develop models for more complex or sophisticated situations than in previous courses.

Cluster	**Build new functions from existing functions.**
M.3HS.42 (*)	Identify the effect on the graph of replacing f(x) by f(x) + k, k f(x), f(kx), and f(x + k) for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them. Instructional Note: Use transformations of functions to find more optimum models as students consider increasingly more complex situations. Note the effect of multiple transformations on a single function and the common effect of each transformation across function types. Include functions defined only by graph.
M.3HS.43 (*)	Find inverse functions. Solve an equation of the form f(x) = c for a simple function f that has an inverse and write an expression for the inverse. (e.g., $f(x) = 2x^3$ or f(x) = (x+1)/(x-1) for x ≠ 1.) Instructional Note: Extend this standard to simple rational, simple radical, and simple exponential functions.

Cluster	**Construct and compare linear, quadratic, and exponential models and solve problems.**
M.3HS.44 (*)	For exponential models, express as a logarithm the solution to $a\,b^{ct} = d$ where a, c, and d are numbers and the base b is 2, 10, or e; evaluate the logarithm using technology. Instructional Note: Consider extending this unit to include the relationship between properties of logarithms and properties of exponents, such as the connection between the properties of exponents and the basic logarithm property that log xy = log x + log y.

Cluster	**Visualize relationships between two dimensional and three-dimensional objects.**
M.3HS.45 (*,^)	Identify the shapes of two-dimensional cross-sections of three dimensional objects and identify three-dimensional objects generated by rotations of two-dimensional objects.

Cluster	**Apply geometric concepts in modeling situations.**
M.3HS.46 (*,^)	Use geometric shapes, their measures and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).
M.3HS.47 (*,^)	Apply concepts of density based on area and volume in modeling situations (e.g., persons per square mile or BTUs per cubic foot).
M.3HS.48 (*,^)	Apply geometric methods to solve design problems (e.g., designing an object or

	structure to satisfy physical constraints or minimize cost and/or working with typographic grid systems based on ratios).

TRADITIONAL PATHWAY

Mathematics – High School Algebra I for 8th Grade

All West Virginia teachers are responsible for classroom instruction that integrates content standards and mathematical habits of mind. Students in this course will focus on five critical units that deepen and extend understanding of linear and exponential relationships by contrasting them with each other and by applying linear models to data that exhibit a linear trend, and students engage in methods for analyzing, solving, and using quadratic functions. Students are introduced to methods for analyzing and using quadratic functions, including manipulating expressions for them, and solving quadratic equations. Students in 8th grade High School Algebra understand and apply the Pythagorean theorem, and use quadratic functions to model and solve problems. Mathematical habits of mind, which should be integrated in these content areas, include: making sense of problems and persevering in solving them, reasoning abstractly and quantitatively; constructing viable arguments and critiquing the reasoning of others; modeling with mathematics; using appropriate tools strategically; attending to precision, looking for and making use of structure; and looking for and expressing regularity in repeated reasoning. Students will continue developing mathematical proficiency in a developmentally-appropriate progressions of standards. Continuing the skill progressions from seventh grade, the following chart represents the mathematical understandings that will be developed:

Relationships between Quantities and Reasoning with Equations	**Linear and Exponential Relationships**
• Solve problems with a wide range of units and solve problems by thinking about units. (e.g., The Trans Alaska Pipeline System is 800 miles long and cost $8 billion to build. Divide one of these numbers by the other. What is the meaning of the answer? Greenland has a population of 56,700 and a land area of 2,175,600 square kilometers. By what factor is the population density of the United States, 80 persons per square mile, larger than the population density of Greenland?)	• Understand contextual relationships of variables and constants. (e.g., Annie is picking apples with her sister. The number of apples in her basket is described by n = 22t + 12, where t is the number of minutes Annie spends picking apples. What do the numbers 22 and 12 tell you about Annie's apple picking?)
Descriptive Statistics	**Expressions and Equations**
• Use linear regression techniques to describe the relationship between quantities and assess the fit of the model. (e.g., Use the high school and university grades for 250 students to create a model that can be used to predict a student's university GPA based on his high school GPA.)	• Interpret algebraic expressions and transforming them purposefully to solve problems. (e.g., In solving a problem about a loan with interest rate r and principal P, seeing the expression $P(1+r)^n$ as a product of P with a factor not depending on P.)
Quadratic Functions and Modeling	
• Solve real-world and mathematical problems by writing and solving nonlinear equations, such as quadratic equations ($ax^2 + bx + c = 0$).	

Numbering of Standards

The following Mathematics Standards will be numbered continuously. The following ranges relate to the clusters found within Mathematics:

Relationships between Quantities and Reasoning with Equations	
Reason quantitatively and use units to solve problems.	Standards 1-3
Interpret the structure of expressions.	Standard 4
Create equations that describe numbers or relationships.	Standards 5-8
Understand solving equations as a process of reasoning and explain the reasoning.	Standard 9
Solve equations and inequalities in one variable.	Standard 10
Linear and Exponential Relationships	
Extend the properties of exponents to rational exponents.	Standards 11-12
Analyze and solve linear equations and pairs of simultaneous linear equations.	Standard 13
Solve systems of equations.	Standards 14-15
Represent and solve equations and inequalities graphically.	Standards 16-18
Define, evaluate and compare functions.	Standards 19-21
Understand the concept of a function and use function notation.	Standards 22-24
Use functions to model relationships between quantities.	Standards 25-26
Interpret functions that arise in applications in terms of a context.	Standards 27-29
Analyze functions using different representations.	Standards 30-31
Build a function that models a relationship between two quantities.	Standards 32-33
Build new functions from existing functions.	Standard 34
Construct and compare linear, quadratic, and exponential models and solve problems.	Standards 35-37
Interpret expressions for functions in terms of the situation they model.	Standard 38
Descriptive Statistics	
Summarize, represent, and interpret data on a single count or measurement variable.	Standards 39-41
Investigate patterns of association in bivariate data.	Standards 42-45
Summarize, represent, and interpret data on two categorical and quantitative variables.	Standards 46-47
Interpret linear models.	Standards 48-50
Expressions and Equations	
Interpret the structure of equations.	Standards 51-52

Write expressions in equivalent forms to solve problems.	Standard 53
Perform arithmetic operations on polynomials.	Standard54
Create equations that describe numbers or relationships.	Standards 55-57
Solve equations and inequalities in one variable.	Standard 58
Solve systems of equations.	Standard 59
Quadratic Functions and Modeling	
Use properties of rational and irrational numbers.	Standard 60
Understand and apply the Pythagorean theorem.	Standards 61-63
Interpret functions that arise in applications in terms of a context.	Standards 64-66
Analyze functions using different representations.	Standards 67-69
Build a function that models a relationship between two quantities.	Standard 70
Build new functions from existing functions.	Standards 71-72
Construct and compare linear, quadratic and exponential models and solve problems.	Standard 73

Relationships between Quantities

Cluster	Reason quantitatively and use units to solve problems
M.A18.1	Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.
M.A18.2	Define appropriate quantities for the purpose of descriptive modeling. Instructional Note: Working with quantities and the relationships between them provides grounding for work with expressions, equations, and functions.
M.A18.3	Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.

Cluster	Interpret the structure of expressions.
M.A18.4	Interpret expressions that represent a quantity in terms of its context. a. Interpret parts of an expression, such as terms, factors, and coefficients. b . Interpret complicated expressions by viewing one or more of their parts as a single entity. For example, interpret $P(1 + r)^n$ as the product of P and a factor not depending on P. Instructional Note: Limit to linear expressions and to exponential expressions with integer exponents.

Cluster	Create equations that describe numbers or relationships.
M.A18.5	Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions. Instructional Note: Limit to linear and exponential equations, and, in the case of exponential equations, limit to situations requiring evaluation of exponential functions at integer inputs.

M.A18.6	Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales. Instructional Note: Limit to linear and exponential equations, and, in the case of exponential equations, limit to situations requiring evaluation of exponential functions at integer inputs.
M.A18.7	Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context. (e.g., Represent inequalities describing nutritional and cost constraints on combinations of different foods.) Instructional Note: Limit to linear equations and inequalities.
M.A18.8	Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. (e.g., Rearrange Ohm's law V = IR to highlight resistance R.) Instructional Note: Limit to formulas with a linear focus.

Cluster	**Understand solving equations as a process of reasoning and explain the reasoning.**
M.A18.9	Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method. Instructional Note: Students should focus on linear equations and be able to extend and apply their reasoning to other types of equations in future units and courses. Students will solve exponential equations in Algebra II.

Cluster	**Solve equations and inequalities in one variable.**
M.A18.10	Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters. Instructional Note: Extend earlier work with solving linear equations to solving linear inequalities in one variable and to solving literal equations that are linear in the variable being solved for. Include simple exponential equations that rely only on application of the laws of exponents, such as $5^x = 125$ or $2^x = 1/16$.

Linear and Exponential Relationships

Cluster	**Extend the properties of exponents to rational exponents.**
M.A18.11	Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents. For example, we define $5^{1/3}$ to be the cube root of 5 because we want $(5^{1/3})^3 = 5^{(1/3)3}$ to hold, so $(5^{1/3})^3$ must equal 5. Instructional Note: Address this standard before discussing exponential functions with continuous domains.
M.A18.12	Rewrite expressions involving radicals and rational exponents using the properties of exponents. Instructional Note: Address this standard before discussing exponential functions with continuous domains.

Cluster	**Analyze and solve linear equations and pairs of simultaneous linear equations.**
M.A18.13	Analyze and solve pairs of simultaneous linear equations.

	a. Understand that solutions to a system of two linear equations in two variables correspond to points of intersection of their graphs, because points of intersection satisfy both equations simultaneously. b. Solve systems of two linear equations in two variables algebraically, and estimate solutions by graphing the equations. Solve simple cases by inspection. For example, 3x + 2y = 5 and 3x + 2y = 6 have no solution because 3x + 2y cannot simultaneously be 5 and 6. c. Solve real-world and mathematical problems leading to two linear equations in two variables. (e.g., Given coordinates for two pairs of points, determine whether the line through the first pair of points intersects the line through the second pair.) Instructional Note: While this content is likely subsumed by M.A18.10, 14, and 15, it could be used for scaffolding instruction to the more sophisticated content found there.

Cluster	**Solve systems of equations**.
M.A18.14	Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions. Instructional Note: Include cases where two equations describe the same line (yielding infinitely many solutions) and cases where two equations describe parallel lines (yielding no solution).
M.A18.15	Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables. Instructional Note: Include cases where two equations describe the same line (yielding infinitely many solutions) and cases where two equations describe parallel lines (yielding no solution).

Cluster	**Represent and solve equations and inequalities graphically.**
M.A18.16	Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line). Instructional Note: Focus on linear and exponential equations and be able to adapt and apply that learning to other types of equations in future courses.
M.A18.17	Explain why the x-coordinates of the points where the graphs of the equations y = f(x) and y = g(x) intersect are the solutions of the equation f(x) = g(x); find the solutions approximately (e.g., using technology to graph the functions, make tables of values or find successive approximations). Include cases where f(x) and/or g(x) are linear, polynomial, rational, absolute value, exponential and logarithmic functions. Instructional Note: Focus on cases where f(x) and g(x) are linear or exponential.
M.A18.18	Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.

Cluster	**Define, evaluate and compare functions.** Instructional Note: While this content is likely subsumed by M.A18.22-24 and M.A18.30a it could be used for scaffolding instruction to the more sophisticated content found there.

M.A18.19	Understand that a function is a rule that assigns to each input exactly one output. The graph of a function is the set of ordered pairs consisting of an input and the corresponding output.
M.A18.20	Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). (e.g., Given a linear function represented by a table of values and a linear function represented by an algebraic expression, determine which function has the greater rate of change.)
M.A18.21	Interpret the equation y = mx + b as defining a linear function, whose graph is a straight line; give examples of functions that are not linear. (e.g., The function $A = s^2$ giving the area of a square as a function of its side length is not linear because its graph contains the points (1,1), (2,4) and (3,9), which are not on a straight line.)

Cluster	**Understand the concept of a function and use function notation.**
M.A18.22	Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then f(x) denotes the output of f corresponding to the input x. The graph of f is the graph of the equation y = f(x). Instructional Note: Students should experience a variety of types of situations modeled by functions. Detailed analysis of any particular class of function at this stage is not advised. Students should apply these concepts throughout their future mathematics courses. Constrain examples to linear functions and exponential functions having integral domains.
M.A18.23	Use function notation, evaluate functions for inputs in their domains and interpret statements that use function notation in terms of a context. Instructional Note: Students should experience a variety of types of situations modeled by functions. Detailed analysis of any particular class of function at this stage is not advised. Students should apply these concepts throughout their future mathematics courses. Constrain examples to linear functions and exponential functions having integral domains.
M.A18.24	Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. For example, the Fibonacci sequence is defined recursively by f(0) = f(1) = 1, f(n+1) = f(n)+ f(n-1) for n ≥ 1. Instructional Note: Students should experience a variety of types of situations modeled by functions. Detailed analysis of any particular class of function at this stage is not advised. Students should apply these concepts throughout their future mathematics courses. Constrain examples to linear functions and exponential functions having integral domains. Draw connection to M.A18.33, which requires students to write arithmetic and geometric sequences.)

Cluster	**Use functions to model relationships between quantities.** Instructional Note: While this content is likely subsumed by M.A18.27and M.A18.32a, it could be used for scaffolding instruction to the more sophisticated content found there.
M.A18.25	Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (x, y) values, including reading these from a table or from a

	graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values.
M.A18.26	Describe qualitatively the functional relationship between two quantities by analyzing a graph (e.g., where the function is increasing or decreasing, linear or nonlinear). Sketch a graph that exhibits the qualitative features of a function that has been described verbally.

Cluster	**Interpret functions that arise in applications in terms of a context.**
M.A18.27	For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity. Instructional Note: Focus on linear and exponential functions.
M.A18.28	Relate the domain of a function to its graph and where applicable, to the quantitative relationship it describes. For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function. Instructional Note: Focus on linear and exponential functions.
M.A18.29	Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph. Instructional Note: Focus on linear functions and intervals for exponential functions whose domain is a subset of the integers. The Quadratic Functions and Modeling unit of this course and Algebra II course will address other function types.

Cluster	**Analyze functions using different representations.**
M.A18.30	Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. a. Graph linear and quadratic functions and show intercepts, maxima, and minima. b. Graph exponential and logarithmic functions, showing intercepts and end behavior and trigonometric functions, showing period, midline and amplitude. Instructional Note: Focus on linear and exponential functions. Include comparisons of two functions presented algebraically. For example, compare the growth of two linear functions, or two exponential functions such as $y = 3^n$ and $y = 100 \cdot 2^n$.
M.A18.31	Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). (e.g., Given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.) Instructional Note: Focus on linear and exponential functions. Include comparisons of two functions presented algebraically. For example, compare the growth of two linear functions, or two exponential functions such as $y = 3^n$ and $y = 100 \cdot 2^n$.

Cluster	**Build a function that models a relationship between two quantities.**
M.A18.32	Write a function that describes a relationship between two quantities. a. Determine an explicit expression, a recursive process, or steps for calculation from a context.

	b. Combine standard function types using arithmetic operations. For example build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model. Instructional Note: Limit to linear and exponential functions.
M.A18.33	Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms. Instructional Note: Connect arithmetic sequences to linear functions and geometric sequences to exponential functions.

Cluster	**Build new functions from existing functions.**
M.A18.34	Identify the effect on the graph of replacing f(x) by f(x) + k, k f(x), f(kx), and f(x + k) for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them. Instructional Note: Focus on vertical translations of graphs of linear and exponential functions. Relate the vertical translation of a linear function to its y-intercept. While applying other transformations to a linear graph is appropriate at this level, it may be difficult for students to identify or distinguish between the effects of the other transformations included in this standard.

Cluster	**Construct and compare linear, quadratic, and exponential models and solve problems.**
M.A18.35	Distinguish between situations that can be modeled with linear functions and with exponential functions. a. Prove that linear functions grow by equal differences over equal intervals; exponential functions grow by equal factors over equal intervals. b. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another. c. Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.
M.A18.36	Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship or two input-output pairs (include reading these from a table).
M.A18.37	Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function. Instructional Note: Limit to comparisons between linear and exponential models.

Cluster	**Interpret expressions for functions in terms of the situation they model.**
M.A18.38	Interpret the parameters in a linear or exponential function in terms of a context. Instructional Note: Limit exponential functions to those of the form $f(x) = b^x + k$.

Descriptive Statistics

Cluster	**Summarize, represent, and interpret data on a single count or measurement variable.**

M.A18.39	Represent data with plots on the real number line (dot plots, histograms, and box plots).
M.A18.40	Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets. Instructional Note: In grades 6 – 7, students describe center and spread in a data distribution. Here they choose a summary statistic appropriate to the characteristics of the data distribution, such as the shape of the distribution or the existence of extreme data points.
M.A18.41	Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers). Instructional Note: In grades 6 – 7, students describe center and spread in a data distribution. Here they choose a summary statistic appropriate to the characteristics of the data distribution, such as the shape of the distribution or the existence of extreme data points.

Cluster	**Investigate patterns of association in bivariate data.** Instructional Note: While this content is likely subsumed by M.A18.47-50, it could be used for scaffolding instruction to the more sophisticated content found there.
M.A18.42	Construct and interpret scatter plots for bivariate measurement data to investigate patterns of association between two quantities. Describe patterns such as clustering, outliers, positive or negative association, linear association, and nonlinear association.
M.A18.43	Know that straight lines are widely used to model relationships between two quantitative variables. For scatter plots that suggest a linear association, informally fit a straight line, and informally assess the model fit by judging the closeness of the data points to the line.
M.A18.44	Use the equation of a linear model to solve problems in the context of bivariate measurement data, interpreting the slope and intercept. (e.g., In a linear model for a biology experiment, interpret a slope of 1.5 cm/hr as meaning that an additional hour of sunlight each day is associated with an additional 1.5 cm in mature plant height.)
M.A18.45	Understand that patterns of association can also be seen in bivariate categorical data by displaying frequencies and relative frequencies in a two-way table. Construct and interpret a two-way table summarizing data on two categorical variables collected from the same subjects. Use relative frequencies calculated for rows or columns to describe possible association between the two variables. (e.g., Collect data from students in your class on whether or not they have a curfew on school nights and whether or not they have assigned chores at home. Is there evidence that those who have a curfew also tend to have chores?)

Cluster	**Summarize, represent, and interpret data on two categorical and quantitative variables.**
M.A18.46	Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal and conditional relative frequencies). Recognize possible associations and trends in the data.
M.A18.47	Represent data on two quantitative variables on a scatter plot, and describe how the variables are related. a. Fit a function to the data; use functions fitted to data to solve problems in the context of the data. Use given functions or choose a function suggested by the context. Instructional Note: Emphasize linear and exponential models.

	b. Informally assess the fit of a function by plotting and analyzing residuals. Instructional Note: Focus should be on situations for which linear models are appropriate, but may be used to preview quadratic functions in the Quadratic Functions and Modeling Unit. c. Fit a linear function for scatter plots that suggest a linear association. Instructional Note: Students take a more sophisticated look at using a linear function to model the relationship between two numerical variables. In addition to fitting a line to data, students assess how well the model fits by analyzing residuals.

Cluster	**Interpret linear models.**
M.A18.48	Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data. Instructional Note: Build on students' work with linear relationships and introduce the correlation coefficient. The focus here is on the computation and interpretation of the correlation coefficient as a measure of how well the data fit the relationship.
M.A18.49	Compute (using technology) and interpret the correlation coefficient of a linear fit. Instructional Note: Build on students' work with linear relationships and introduce the correlation coefficient. The focus here is on the computation and interpretation of the correlation coefficient as a measure of how well the data fit the relationship.
M.A18.50	Distinguish between correlation and causation. Instructional Note: The important distinction between a statistical relationship and a cause-and-effect relationship arises here.

Expressions and Equations

Cluster	**Interpret the structure of equations.**
M.A18.51	Interpret expressions that represent a quantity in terms of its context. a. Interpret parts of an expression, such as terms, factors, and coefficients. b. Interpret complicated expressions by viewing one or more of their parts as a single entity. For example, interpret $P(1 + r)^n$ as the product of P and a factor not depending on P. Instructional Note: Focus on quadratic and exponential expressions. For M.A18.51b, exponents are extended from integer found in the unit on Relationships between Quantities to rational exponents focusing on those that represent square roots and cube roots.
M.A18.52	Use the structure of an expression to identify ways to rewrite it. For example, see $x^4 - y^4$ as $(x^2)^2 - (y^2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2 - y^2)(x^2 + y^2)$.

Cluster	**Write expressions in equivalent forms to solve problems.**
M.A18.53	Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. a. Factor a quadratic expression to reveal the zeros of the function it defines. b. Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines. c. Use the properties of exponents to transform expressions for exponential functions. For example the expression 1.15^t can be rewritten as $(1.15^{1/12})^{12t} \approx$

	1.012^{12t} to reveal the approximate equivalent monthly interest rate if the annual rate is 15%. Instructional Note: It is important to balance conceptual understanding and procedural fluency in work with equivalent expressions. For example, development of skill in factoring and completing the square goes hand-in-hand with understanding what different forms of a quadratic expression reveal.

Cluster	**Perform arithmetic operations on polynomials.**
M.A18.54	Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials. Instructional Note: Focus on polynomial expressions that simplify to forms that are linear or quadratic in a positive integer power of x.

Cluster	**Create equations that describe numbers or relationships.**
M.A18.55	Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions. Instructional Note: Extend work on linear and exponential equations in the unit on Relationships between Quantities to include quadratic equations.
M.A18.56	Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales. Instructional Note: Extend work on linear and exponential equations in the unit on Relationships between Quantities to include quadratic equations.
M.A18.57	Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. (e.g., Rearrange Ohm's law V = IR to highlight resistance R.) Instructional Note: Extend work on linear and exponential equations in the unit on Relationships between Quantities to include quadratic equations. Extend M.A18.57 to formulas involving squared variables.

Cluster	**Solve equations and inequalities in one variable.**
M.A18.58	Solve quadratic equations in one variable. a. Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x – p)^2 = q$ that has the same solutions. Derive the quadratic formula from this form. b. Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as a ± bi for real numbers a and b. Instructional Note: Students should learn of the existence of the complex number system, but will not solve quadratics with complex solutions until Algebra II.

Cluster	**Solve systems of equations.**
M.A18.59	Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically. (e.g., Find the points of intersection between the line $y = –3x$ and the circle $x^2 + y^2 = 3$.) Instructional Note: Include systems consisting of

	one linear and one quadratic equation. Include systems that lead to work with fractions. For example, finding the intersections between $x^2 + y^2 = 1$ and $y = (x+1)/2$ leads to the point (3/5, 4/5) on the unit circle, corresponding to the Pythagorean triple $3^2 + 4^2 = 5^2$.

Quadratic Functions and Modeling

Cluster	Use properties of rational and irrational numbers.
M.A18.60	Explain why the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational. Instructional Note: Connect to physical situations (e.g., finding the perimeter of a square of area 2).

Cluster	Understand and apply the Pythagorean theorem.
M.A18.61	Explain a proof of the Pythagorean Theorem and its converse
M.A18.62	Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions. Instructional Note: Discuss applications of the Pythagorean theorem and its connections to radicals, rational exponents, and irrational numbers.
M.A18.63	Apply the Pythagorean Theorem to find the distance between two points in a coordinate system. Instructional Note: Discuss applications of the Pythagorean theorem and its connections to radicals, rational exponents, and irrational numbers.

Cluster	Interpret functions that arise in applications in terms of a context.
M.A18.64	For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity. Instructional Note: Focus on quadratic functions; compare with linear and exponential functions studies in the unit on Linear and Exponential Functions.
M.A18.65	Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function. Instructional Note: Focus on quadratic functions; compare with linear and exponential functions studies in the unit on Linear and Exponential Functions.
M.A18.66	Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph. Instructional Note: Focus on quadratic functions; compare with linear and exponential functions studies in the unit on Linear and Exponential Functions.

Cluster	Analyze functions using different representations.
M.A18.67	Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. a. Graph linear and quadratic functions and show intercepts, maxima, and minima.

	b. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions. Instructional Note: Compare and contrast absolute value, step and piecewise-defined functions with linear, quadratic, and exponential functions. Highlight issues of domain, range, and usefulness when examining piecewise-defined functions. Extend work with quadratics to include the relationship between coefficients and roots, and that once roots are known, a quadratic function can be factored.
M.A18.68	Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. a. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context. b. Use the properties of exponents to interpret expressions for exponential functions. For example, identify percent rate of change in functions such as $y = (1.02)^t$, $y = (0.97)^t$, $y = (1.01)^{12t}$, $y = (1.2)^{t/10}$, and classify them as representing exponential growth or decay. Instructional Note: Extend work with quadratics to include the relationship between coefficients and roots, and that once roots are known, a quadratic function can be factored. This unit, and in particular in M.A18.68b, extends the work begun in Unit 2 on exponential functions with integral exponents.
M.A18.69	Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). (e.g., Given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.) Instructional Note: Extend work with quadratics to include the relationship between coefficients and roots, and that once roots are known, a quadratic function can be factored.

Cluster	**Build a function that models a relationship between two quantities.**
M.A18.70	Write a function that describes a relationship between two quantities. a. Determine an explicit expression, a recursive process, or steps for calculation from a context. b. Combine standard function types using arithmetic operations. For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model. Instructional Note: Focus on situations that exhibit a quadratic relationship.

Cluster	**Build new functions from existing functions.**
M.A18.71	Identify the effect on the graph of replacing f(x) by f(x) + k, k f(x), f(kx), and f(x + k) for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them. Instructional Note: Focus on quadratic functions, and consider including absolute value functions.
M.A18.72	Find inverse functions.

	a. Solve an equation of the form f(x) = c for a simple function f that has an inverse and write an expression for the inverse. For example, $f(x) = 2\,x^3$ or f(x) = (x+1)/(x-1) for x ≠ 1. Instructional Note: Focus on linear functions but consider simple situations where the domain of the function must be restricted in order for the inverse to exist, such as $f(x) = x^2$, x > 0.

Cluster	**Construct and compare linear, quadratic and exponential models and solve problems.**
M.A18.73	Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function. Instructional Note: Compare linear and exponential growth to growth of quadratic growth.

Mathematics – High School Algebra I

All West Virginia teachers are responsible for classroom instruction that integrates content standards and mathematical habits of mind. Students in this course will focus on five critical units that deepen and extend understanding of linear and exponential relationships by contrasting them with each other and by applying linear models to data that exhibit a linear trend, and students engage in methods for analyzing, solving, and using quadratic functions. Mathematical habits of mind, which should be integrated in these content areas, include: making sense of problems and persevering in solving them, reasoning abstractly and quantitatively; constructing viable arguments and critiquing the reasoning of others; modeling with mathematics; using appropriate tools strategically; attending to precision, looking for and making use of structure; and looking for and expressing regularity in repeated reasoning. Students will continue developing mathematical proficiency in a developmentally-appropriate progressions of standards. Continuing the skill progressions from previous courses, the following chart represents the mathematical understandings that will be developed:

Relationships between Quantities and Reasoning with Equations	Linear and Exponential Relationships
• Solve problems with a wide range of units and solve problems by thinking about units. (e.g., The Trans Alaska Pipeline System is 800 miles long and cost $8 billion to build. Divide one of these numbers by the other. What is the meaning of the answer? Greenland has a population of 56,700 and a land area of 2,175,600 square kilometers. By what factor is the population density of the United States, 80 persons per square mile, larger than the population density of Greenland?)	• Understand contextual relationships of variables and constants. (e.g., Annie is picking apples with her sister. The number of apples in her basket is described by n = 22t + 12, where t is the number of minutes Annie spends picking apples. What do the numbers 22 and 12 tell you about Annie's apple picking?)
Descriptive Statistics	**Expressions and Equations**
• Use linear regression techniques to describe the relationship between quantities and assess the fit of the model. (e.g., Use the high school and university grades for 250 students to create a model that can be used to predict a student's university GPA based on his high school GPA.)	• Interpret algebraic expressions and transform them purposefully to solve problems. (e.g., In solving a problem about a loan with interest rate r and principal P, seeing the expression $P(1+r)^n$ as a product of P with a factor not depending on P.)
Quadratic Functions and Modeling	
• Solve real-world and mathematical problems by writing and solving nonlinear equations, such as quadratic equations ($ax^2 + bx + c = 0$).	

Numbering of Standards

The following Mathematics Standards will be numbered continuously. The following ranges relate to the clusters found within Mathematics:

Relationships between Quantities and Reasoning with Equations	
Reason quantitatively and use units to solve problems.	Standards 1-3
Interpret the structure of expressions.	Standard 4
Create equations that describe numbers or relationships.	Standards 5-8
Understand solving equations as a process of reasoning and explain the reasoning.	Standard 9
Solve equations and inequalities in one variable.	Standard 10
Linear and Exponential Relationships	
Extend the properties of exponents to rational exponents.	Standards 11-12
Solve systems of equations.	Standards 13-14
Represent and solve equations and inequalities graphically.	Standards 15-17
Understand the concept of a function and use function notation.	Standards 18-20
Interpret functions that arise in applications in terms of a context.	Standards 21-23
Analyze functions using different representations.	Standards 24-25
Build a function that models a relationship between two quantities.	Standards 26-27
Build new functions from existing functions.	Standards 28
Construct and compare linear, quadratic, and exponential models and solve problems.	Standards 29-31
Interpret expressions for functions in terms of the situation they model.	Standard 32
Descriptive Statistics	
Summarize, represent, and interpret data on a single count or measurement variable.	Standards 33-35
Summarize, represent, and interpret data on two categorical and quantitative variables.	Standards 36-37
Interpret linear models.	Standards 38-40
Expressions and Equations	
Interpret the structure of equations.	Standards 41-42
Write expressions in equivalent forms to solve problems.	Standard 43
Perform arithmetic operations on polynomials.	Standard 44
Create equations that describe numbers or relationships.	Standards 45-47
Solve equations and inequalities in one variable.	Standard 48
Solve systems of equations.	Standard 49
Quadratic Functions and Modeling	
Use properties of rational and irrational	Standard 50

numbers.	
Interpret functions that arise in applications in terms of a context.	Standards 51-53
Analyze functions using different representations.	Standards 54-56
Build a function that models a relationship between two quantities.	Standards 57
Build new functions from existing functions.	Standard 58-59
Construct and compare linear, quadratic and exponential models and solve problems.	Standard 60

Relationships between Quantities and Reasoning with Equations

Cluster	**Reason quantitatively and use units to solve problems.**
M.A1HS.1	Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.
M.A1HS.2	Define appropriate quantities for the purpose of descriptive modeling. Instructional Note: Working with quantities and the relationships between them provides grounding for work with expressions, equations, and functions.
M.A1HS.3	Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.

Cluster	**Interpret the structure of expressions.**
M.A1HS.4	Interpret expressions that represent a quantity in terms of its context. a. Interpret parts of an expression, such as terms, factors, and coefficients. b. Interpret complicated expressions by viewing one or more of their parts as a single entity. (e.g., Interpret $P(1 + r)^n$ as the product of P and a factor not depending on P. Instructional Note: Limit to linear expressions and to exponential expressions with integer exponents.

Cluster	**Create equations that describe numbers or relationships.**
M.A1HS.5	Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions. Instructional Note: Limit to linear and exponential equations, and, in the case of exponential equations, limit to situations requiring evaluation of exponential functions at integer inputs.
M.A1HS.6	Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales. Instructional Note: Limit to linear and exponential equations, and, in the case of exponential equations, limit to situations requiring evaluation of exponential functions at integer inputs.
M.A1HS.7	Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context. (e.g., Represent inequalities describing nutritional and cost constraints on combinations of different foods.) Instructional Note: Limit to linear equations and inequalities.

M.A1HS.8	Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. (e.g., Rearrange Ohm's law V = IR to highlight resistance R.) Instructional Note: Limit to formulas with a linear focus.

Cluster	**Understand solving equations as a process of reasoning and explain the reasoning.**
M.A1HS.9	Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method. Instructional Note: Students should focus on and master linear equations and be able to extend and apply their reasoning to other types of equations in future courses. Students will solve exponential equations with logarithms in Algebra II.

Cluster	**Solve equations and inequalities in one variable.**
M.A1HS.10	Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters. Instructional Note: Extend earlier work with solving linear equations to solving linear inequalities in one variable and to solving literal equations that are linear in the variable being solved for. Include simple exponential equations that rely only on application of the laws of exponents, such as $5^x = 125$ or $2^x = 1/16$.

Linear and Exponential Relationships

Cluster	**Extend the properties of exponents to rational exponents.**
M.A1HS.11	Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents. (e.g., We define $5^{1/3}$ to be the cube root of 5 because we want $(5^{1/3})^3 = 5^{(1/3)3}$ to hold, so $(5^{1/3})^3$ must equal 5.) Instructional Note: Address this standard before discussing exponential functions with continuous domains.
M.A1HS.12	Rewrite expressions involving radicals and rational exponents using the properties of exponents. Instructional Note: Address this standard before discussing exponential functions with continuous domains.

Cluster	**Solve systems of equations**.
M.A1HS.13	Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions.
M.A1HS.14	Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables. Instructional Note: Build on student experiences graphing and solving systems of linear equations from middle school to focus on justification of the methods used. Include cases where the two equations describe the same line (yielding infinitely many solutions) and cases where two equations describe parallel lines (yielding no solution); connect to standards in Geometry which require students to prove the slope criteria for parallel lines.

Cluster	**Represent and solve equations and inequalities graphically.**

M.A1HS.15	Recognize that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line). Instructional Note: Focus on linear and exponential equations and be able to adapt and apply that learning to other types of equations in future courses.
M.A1HS.16	Explain why the x-coordinates of the points where the graphs of the equations y = f(x) and y = g(x) intersect are the solutions of the equation f(x) = g(x); find the solutions approximately (e.g., using technology to graph the functions, make tables of values or find successive approximations). Include cases where f(x) and/or g(x) are linear, polynomial, rational, absolute value, exponential and logarithmic functions. Instructional Note: Focus on cases where f(x) and g(x) are linear or exponential.
M.A1HS.17	Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.

Cluster	**Understand the concept of a function and use function notation.**
M.A1HS.18	Recognize that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then f(x) denotes the output of f corresponding to the input x. The graph of f is the graph of the equation y = f(x). Instructional Note: Students should experience a variety of types of situations modeled by functions. Detailed analysis of any particular class of function at this stage is not advised. Students should apply these concepts throughout their future mathematics courses. Draw examples from linear functions and exponential functions having integral domains.
M.A1HS.19	Use function notation, evaluate functions for inputs in their domains and interpret statements that use function notation in terms of a context. Instructional Note: Students should experience a variety of types of situations modeled by functions. Detailed analysis of any particular class of function at this stage is not advised. Students should apply these concepts throughout their future mathematics courses. Draw examples from linear functions and exponential functions having integral domains.
M.A1HS.20	Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. (e.g., The Fibonacci sequence is defined recursively by f(0) = f(1) = 1, f(n+1) = f(n)+ f(n-1) for n ≥ 1. Instructional Note: Students should experience a variety of types of situations modeled by functions. Detailed analysis of any particular class of function at this stage is not advised. Students should apply these concepts throughout their future mathematics courses. Draw examples from linear functions and exponential functions having integral domains. Draw connection to M.A1HS.27, which requires students to write arithmetic and geometric sequences. Emphasize arithmetic and geometric sequences as examples of linear and exponential functions.

Cluster	**Interpret functions that arise in applications in terms of a context.**
M.A1HS.21	For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative

	maximums and minimums; symmetries; end behavior; and periodicity. Instructional Note: Focus on linear and exponential functions.
M.A1HS.22	Relate the domain of a function to its graph and where applicable, to the quantitative relationship it describes. (e.g., If the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.) Instructional Note: Focus on linear and exponential functions.
M.A1HS.23	Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph. Instructional Note: Focus on linear functions and exponential functions whose domain is a subset of the integers. The Unit on Quadratic Functions and Modeling in this course and the Algebra II course address other types of functions.

Cluster	Analyze functions using different representations.
M.A1HS.24	Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. a. Graph linear and quadratic functions and show intercepts, maxima, and minima. b. Graph exponential and logarithmic functions, showing intercepts and end behavior and trigonometric functions, showing period, midline and amplitude. Instructional Note: Focus on linear and exponential functions. Include comparisons of two functions presented algebraically. For example, compare the growth of two linear functions, or two exponential functions such as $y = 3^n$ and $y = 100^{2n}$)
M.A1HS.25	Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). (e.g., Given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.) Instructional Note: Focus on linear and exponential functions. Include comparisons of two functions presented algebraically. For example, compare the growth of two linear functions, or two exponential functions such as $y = 3^n$ and $y = 100^{2n}$)

Cluster	Build a function that models a relationship between two quantities.
M.A1HS.26	Write a function that describes a relationship between two quantities. a. Determine an explicit expression, a recursive process, or steps for calculation from a context. b. Combine standard function types using arithmetic operations. (e.g., Build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.) Instructional Note: Limit to linear and exponential functions.
M.A1HS.27	Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms. Instructional Note: Limit to linear and exponential functions. Connect arithmetic sequences to linear functions and geometric sequences to exponential functions.

Cluster	Build new functions from existing functions.
M.A1HS.28	Identify the effect on the graph of replacing f(x) by f(x) + k, k f(x), f(kx), and f(x + k) for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using

	technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them. Instructional Note: Focus on vertical translations of graphs of linear and exponential functions. Relate the vertical translation of a linear function to its y-intercept. While applying other transformations to a linear graph is appropriate at this level, it may be difficult for students to identify or distinguish between the effects of the other transformations included in this standard.

Cluster	Construct and compare linear, quadratic, and exponential models and solve problems.
M.A1HS.29	Distinguish between situations that can be modeled with linear functions and with exponential functions. a. Prove that linear functions grow by equal differences over equal intervals; exponential functions grow by equal factors over equal intervals. b. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another. c. Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.
M.A1HS.30	Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship or two input-output pairs (include reading these from a table). Instructional Note: In constructing linear functions, draw on and consolidate previous work in Grade 8 on finding equations for lines and linear functions.
M.A1HS.31	Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function. Instructional Note: Limit to comparisons between exponential and linear models.

Cluster	Interpret expressions for functions in terms of the situation they model.
M.A1HS.32	Interpret the parameters in a linear or exponential function in terms of a context. Instructional Note: Limit exponential functions to those of the form $f(x) = b^x + k$.

Descriptive Statistics

Cluster	Summarize, represent, and interpret data on a single count or measurement variable.
M.A1HS.33	Represent data with plots on the real number line (dot plots, histograms, and box plots).
M.A1HS.34	Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets. Instructional Note: In grades 6 – 8, students describe center and spread in a data distribution. Here they choose a summary statistic appropriate to the characteristics of the data distribution, such as the shape of the distribution or the existence of extreme data points.
M.A1HS.35	Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers). Instructional Note: In grades 6 – 8, students describe center and spread in a data distribution. Here they choose a summary statistic appropriate to the characteristics of the data distribution, such as the shape of the distribution or the existence of extreme data points.

Cluster	**Summarize, represent, and interpret data on two categorical and quantitative variables.**
M.A1HS.36	Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal and conditional relative frequencies). Recognize possible associations and trends in the data.
M.A1HS.37	Represent data on two quantitative variables on a scatter plot, and describe how the variables are related. a. Fit a function to the data; use functions fitted to data to solve problems in the context of the data. Use given functions or choose a function suggested by the context. Emphasize linear and exponential models. b. Informally assess the fit of a function by plotting and analyzing residuals. Instructional Note: Focus should be on situations for which linear models are appropriate. c. Fit a linear function for scatter plots that suggest a linear association. Instructional Note: Students take a more sophisticated look at using a linear function to model the relationship between two numerical variables. In addition to fitting a line to data, students assess how well the model fits by analyzing residuals.

Cluster	**Interpret linear models.**
M.A1HS.38	Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data. Instructional Note: Build on students' work with linear relationships in eighth grade and introduce the correlation coefficient. The focus here is on the computation and interpretation of the correlation coefficient as a measure of how well the data fit the relationship.
M.A1HS.39	Compute (using technology) and interpret the correlation coefficient of a linear fit. Instructional Note: Build on students' work with linear relationships in eighth grade and introduce the correlation coefficient. The focus here is on the computation and interpretation of the correlation coefficient as a measure of how well the data fit the relationship.
M.A1HS.40	Distinguish between correlation and causation. Instructional Note: The important distinction between a statistical relationship and a cause-and-effect relationship is the focus.

Expressions and Equations

Cluster	**Interpret the structure of equations.**
M.A1HS.41	Interpret expressions that represent a quantity in terms of its context. a. Interpret parts of an expression, such as terms, factors, and coefficients. b. Interpret complicated expressions by viewing one or more of their parts as a single entity. For example, interpret $P(1 + r)^n$ as the product of P and a factor not depending on P. Instructional Note: Exponents are extended from the integer exponents found in the unit on Relationships between Quantities and Reasoning with Equations to rational exponents focusing on those that represent square or cube roots. Instructional Note: Focus on quadratic and exponential expressions.

M.A1HS.42	Use the structure of an expression to identify ways to rewrite it. For example, see $x^4 - y^4$ as $(x^2)^2 - (y^2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2 - y^2)(x^2 + y^2)$. Instructional Note: Focus on quadratic and exponential expressions.

Cluster	**Write expressions in equivalent forms to solve problems.**
M.A1HS.43	Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. a. Factor a quadratic expression to reveal the zeros of the function it defines. b. Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines. c. Use the properties of exponents to transform expressions for exponential functions. For example the expression 1.15^t can be rewritten as $(1.15^{1/12})^{12t} \approx 1.012^{12t}$ to reveal the approximate equivalent monthly interest rate if the annual rate is 15%. Instructional Note: It is important to balance conceptual understanding and procedural fluency in work with equivalent expressions. For example, development of skill in factoring and completing the square goes hand-in-hand with understanding what different forms of a quadratic expression reveal.

Cluster	**Perform arithmetic operations on polynomials.**
M.A1HS.44	Recognize that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials. Instructional Note: Focus on polynomial expressions that simplify to forms that are linear or quadratic in a positive integer power of x.

Cluster	**Create equations that describe numbers or relationships.**
M.A1HS.45	Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions. Instructional Note: Extend work on linear and exponential equations in the Relationships between Quantities and Reasoning with Equations unit to quadratic equations.
M.A1HS.46	Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales. Instructional Note: Extend work on linear and exponential equations in the Relationships between Quantities and Reasoning with Equations unit to quadratic equations.
M.A1HS.47	Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. (e.g., Rearrange Ohm's law V = IR to highlight resistance R. Instructional Note: Extend work on linear and exponential equations in the Relationships between Quantities and Reasoning with Equations unit to quadratic equations. Extend this standard to formulas involving squared variables.

Cluster	**Solve equations and inequalities in one variable.**
M.A1HS.48	Solve quadratic equations in one variable. a. Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x - p)^2 = q$ that has the same solutions. Derive the quadratic formula from this form. b. Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots,

	completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as a ± bi for real numbers a and b. Instructional Note: Students should learn of the existence of the complex number system, but will not solve quadratics with complex solutions until Algebra II.

Cluster	Solve systems of equations.
M.A1HS.49	Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically. For example, find the points of intersection between the line $y = –3x$ and the circle $x^2 + y^2 = 3$. Instructional Note: Include systems consisting of one linear and one quadratic equation. Include systems that lead to work with fractions. For example, finding the intersections between $x^2 + y^2 = 1$ and $y = (x+1)/2$ leads to the point (3/5, 4/5) on the unit circle, corresponding to the Pythagorean triple $3^2 + 4^2 = 5^2$.

Quadratic Functions and Modeling

Cluster	Use properties of rational and irrational numbers.
M.A1HS.50	Explain why the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational. Instructional Note: Connect to physical situations (e.g., finding the perimeter of a square of area 2).

Cluster	Interpret functions that arise in applications in terms of a context.
M.A1HS.51	For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity. Instructional Note: Focus on quadratic functions; compare with linear and exponential functions studied in the Unit on Linear and Exponential Relationships.
M.A1HS.52	Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function. Instructional Note: Focus on quadratic functions; compare with linear and exponential functions studied in the Unit on Linear and Exponential Relationships.
M.A1HS.53	Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph. Instructional Note: Focus on quadratic functions; compare with linear and exponential functions studied in the Unit on Linear and Exponential Relationships.

Cluster	Analyze functions using different representations.
M.A1HS.54	Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.

	a. Graph linear and quadratic functions and show intercepts, maxima, and minima. b. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions. Instructional Note: Compare and contrast absolute value, step and piecewise-defined functions with linear, quadratic, and exponential functions. Highlight issues of domain, range, and usefulness when examining piecewise-defined functions. Extend work with quadratics to include the relationship between coefficients and roots, and that once roots are known, a quadratic equation can be factored.
M.A1HS.55	Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. a. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context. b. Use the properties of exponents to interpret expressions for exponential functions. For example, identify percent rate of change in functions such as $y = (1.02)^t$, $y = (0.97)^t$, $y = (1.01)^{12t}$, $y = (1.2)^{t/10}$, and classify them as representing exponential growth or decay. Instructional Note: Extend work with quadratics to include the relationship between coefficients and roots, and that once roots are known, a quadratic equation can be factored. This standard extends the work begun in the Linear and Exponential Relationships unit on exponential functions with integer exponents.
M.A1HS.56	Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum. Instructional Note: Highlight issues of domain, range, and usefulness when examining piecewise-defined functions. Extend work with quadratics to include the relationship between coefficients and roots, and that once roots are known, a quadratic equation can be factored.

Cluster	**Build a function that models a relationship between two quantities.**
M.A1HS.57	Write a function that describes a relationship between two quantities. a. Determine an explicit expression, a recursive process, or steps for calculation from a context. b. Combine standard function types using arithmetic operations. For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model. Instructional Note: Focus on situations that exhibit a quadratic relationship.

Cluster	**Build new functions from existing functions.**
M.A1HS.58	Identify the effect on the graph of replacing f(x) by f(x) + k, k f(x), f(kx), and f(x + k) for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them. Instructional Note: Focus on quadratic functions, and consider including absolute value functions.

M.A1HS.59	Find inverse functions. Solve an equation of the form f(x) = c for a simple function f that has an inverse and write an expression for the inverse. For example, $f(x) = 2\ x^3$ or $f(x) = (x+1)/(x-1)$ for $x \neq 1$. Instructional Note: Focus on linear functions but consider simple situations where the domain of the function must be restricted in order for the inverse to exist, such as $f(x) = x^2$, $x > 0$.

Cluster	**Construct and compare linear, quadratic and exponential models and solve problems.**
M.A1HS.60	Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function. Instructional Note: Compare linear and exponential growth to quadratic growth.

Mathematics - High School Geometry

All West Virginia teachers are responsible for classroom instruction that integrates content standards and mathematical habits of mind. Students in this course will explore more complex geometric situations and deepen their explanations of geometric relationships, moving towards formal mathematical arguments. Important differences exist between this Geometry course and the historical approach taken in Geometry classes. For example, transformations are emphasized early in this course. Mathematical habits of mind, which should be integrated in these content areas, include: making sense of problems and persevering in solving them, reasoning abstractly and quantitatively; constructing viable arguments and critiquing the reasoning of others; modeling with mathematics; using appropriate tools strategically; attending to precision, looking for and making use of structure; and looking for and expressing regularity in repeated reasoning. Students will continue developing mathematical proficiency in a developmentally-appropriate progressions of standards. Continuing the skill progressions from previous courses, the following chart represents the mathematical understandings that will be developed:

Congruence, Proof, and Constructions	**Similarity, Proof, and Trigonometry**
• Prove theorems about triangles and other figures (e.g., that the sum of the measures of the angles in a triangle is 180°). • Given a transformation, work backwards to discover the sequence that led to the transformation. • Given two quadrilaterals that are reflections of each other, find the line of that reflection.	• Apply knowledge of trigonometric ratios and the Pythagorean Theorem to determine distances in realistic situations. (e.g., Determine heights of inaccessible objects using various instruments, such as clinometers, hypsometers, transits, etc.)
Extending to Three Dimensions	**Connecting Algebra and Geometry Through Coordinates**
• Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone.	• Use a rectangular coordinate system and build on understanding of the Pythagorean Theorem to find distances. (e.g., Find the area and perimeter of a real-world shape using a coordinate grid and Google Earth.) • Analyze the triangles and quadrilaterals on the coordinate plane to determine their properties. (e.g., Determine whether a given quadrilateral is a rectangle).
Circles With and Without Coordinates	**Applications of Probability**
• Use coordinates and equations to describe geometric properties algebraically. (e.g., Write the equation for a circle in the plane with specified center and radius.) **Modeling with Geometry** • Analyze real-world situations using mathematics to understand the situation better and optimize, troubleshoot, or make an informed decision (e.g., estimate water and food needs in a disaster area, or use	• Work with probability and using ideas from probability in everyday situations. (e.g., Compare the chance that a person who smokes will develop lung cancer to the chance that a person who develops lung cancer smokes.)

volume formulas and graphs to find an optimal size for an industrial package).	

Numbering of Standards

The following Mathematics Standards will be numbered continuously. The following ranges relate to the clusters found within Mathematics:

Congruence, Proof, and Constructions	
Experiment with transformations in the plane.	Standards 1-5
Understand congruence in terms of rigid motions.	Standards 6-8
Prove geometric theorems.	Standards 9-11
Make geometric constructions.	Standards 12-13
Similarity, Proof, and Trigonometry	
Understand similarity in terms of similarity transformations.	Standards 14-16
Prove theorems involving similarity.	Standards 17-18
Define trigonometric ratios and solve problems involving right triangles.	Standards 19-21
Apply trigonometry to general triangles.	Standards 22-24
Extending to Three Dimensions	
Explain volume formulas and use them to solve problems.	Standards 25-26
Visualize the relation between two dimensional and three-dimensional objects.	Standard 27
Apply geometric concepts in modeling situations.	Standard 28
Connecting Algebra and Geometry Through Coordinates	
Use coordinates to prove simple geometric theorems algebraically.	Standards 29-32
Translate between the geometric description and the equation for a conic section.	Standard 33
Circles With and Without Coordinates	
Understand and apply theorems about circles.	Standards 34-37
Find arc lengths and areas of sectors of circles.	Standard 38
Translate between the geometric description and the equation for a conic section.	Standard 39
Use coordinates to prove simple geometric theorems algebraically.	Standard 40
Apply geometric concepts in modeling situations.	Standard 41
Applications of Probability	
Understand independence and conditional probability and use them to interpret data.	Standards 42-46
Use the rules of probability to compute probabilities of compound events in a uniform probability model.	Standards 47-50
Use probability to evaluate outcomes of decisions.	Standards 51-52

Modeling with Geometry	
Visualize relationships between two dimensional and three-dimensional objects and apply geometric concepts in modeling situations.	Standards 53-55

Congruence, Proof and Constructions

Cluster	Experiment with transformations in the plane.
M.GHS.1	Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.
M.GHS.2	Represent transformations in the plane using, for example, transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not (e.g., translation versus horizontal stretch). Instructional Note: Build on student experience with rigid motions from earlier grades. Point out the basis of rigid motions in geometric concepts, (e.g., translations move points a specified distance along a line parallel to a specified line; rotations move objects along a circular arc with a specified center through a specified angle).
M.GHS.3	Given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and reflections that carry it onto itself. Instructional Note: Build on student experience with rigid motions from earlier grades. Point out the basis of rigid motions in geometric concepts, (e.g., translations move points a specified distance along a line parallel to a specified line; rotations move objects along a circular arc with a specified center through a specified angle).
M.GHS.4	Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments. Instructional Note: Build on student experience with rigid motions from earlier grades. Point out the basis of rigid motions in geometric concepts (e.g., translations move points a specified distance along a line parallel to a specified line; rotations move objects along a circular arc with a specified center through a specified angle).
M.GHS.5	Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using, for example, graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another. Instructional Note: Build on student experience with rigid motions from earlier grades. Point out the basis of rigid motions in geometric concepts, (e.g., translations move points a specified distance along a line parallel to a specified line; rotations move objects along a circular arc with a specified center through a specified angle)

Cluster	Understand congruence in terms of rigid motions.
M.GHS.6	Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent. Instructional Note: Rigid motions are at the foundation of the definition of congruence. Students reason from the basic properties of rigid motions (that they preserve distance and angle), which are assumed without proof. Rigid motions and their assumed properties

	can be used to establish the usual triangle congruence criteria, which can then be used to prove other theorems.
M.GHS.7	Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent. Instructional Note: Rigid motions are at the foundation of the definition of congruence. Students reason from the basic properties of rigid motions (that they preserve distance and angle), which are assumed without proof. Rigid motions and their assumed properties can be used to establish the usual triangle congruence criteria, which can then be used to prove other theorems.
M.GHS.8	Explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow from the definition of congruence in terms of rigid motions. Instructional Note: Rigid motions are at the foundation of the definition of congruence. Students reason from the basic properties of rigid motions (that they preserve distance and angle), which are assumed without proof. Rigid motions and their assumed properties can be used to establish the usual triangle congruence criteria, which can then be used to prove other theorems.

Cluster	**Prove geometric theorems.**
M.GHS.9	Prove theorems about lines and angles. Theorems include: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent; points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints. Instructional Note: Encourage multiple ways of writing proofs, such as in narrative paragraphs, using flow diagrams, in two-column format, and using diagrams without words. Students should be encouraged to focus on the validity of the underlying reasoning while exploring a variety of formats for expressing that reasoning.
M.GHS.10	Prove theorems about triangles. Theorems include: measures of interior angles of a triangle sum to 180°; base angles of isosceles triangles are congruent; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point. Instructional Note: Encourage multiple ways of writing proofs, such as in narrative paragraphs, using flow diagrams, in two-column format, and using diagrams without words. Students should be encouraged to focus on the validity of the underlying reasoning while exploring a variety of formats for expressing that reasoning. Implementation of this standard may be extended to include concurrence of perpendicular bisectors and angle bisectors as preparation for M.GHS.36.
M.GHS.11	Prove theorems about parallelograms. Theorems include: opposite sides are congruent, opposite angles are congruent, the diagonals of a parallelogram bisect each other, and conversely, rectangles are parallelograms with congruent diagonals. Instructional Note: Encourage multiple ways of writing proofs, such as in narrative paragraphs, using flow diagrams, in two-column format, and using diagrams without words. Students should be encouraged to focus on the validity of the underlying reasoning while exploring a variety of formats for expressing that reasoning.

Cluster	**Make geometric constructions.**
M.GHS.12	Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.). Copying a segment; copying an angle; bisecting a segment; bisecting an angle;

	constructing perpendicular lines, including the perpendicular bisector of a line segment; and constructing a line parallel to a given line through a point not on the line. Instructional Note: Build on prior student experience with simple constructions. Emphasize the ability to formalize and explain how these constructions result in the desired objects. Some of these constructions are closely related to previous standards and can be introduced in conjunction with them.
M.GHS.13	Construct an equilateral triangle, a square, and a regular hexagon inscribed in a circle. Instructional Note: Build on prior student experience with simple constructions. Emphasize the ability to formalize and explain how these constructions result in the desired objects. Some of these constructions are closely related to previous standards and can be introduced in conjunction with them.

Similarity, Proof, and Trigonometry

Cluster	**Understand similarity in terms of similarity transformations.**
M.GHS.14	Verify experimentally the properties of dilations given by a center and a scale factor. a. A dilation takes a line not passing through the center of the dilation to a parallel line, and leaves a line passing through the center unchanged. b. The dilation of a line segment is longer or shorter in the ratio given by the scale factor.
M.GHS.15	Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides.
M.GHS.16	Use the properties of similarity transformations to establish the AA criterion for two triangles to be similar.

Cluster	**Prove theorems involving similarity.**
M.GHS.17	Prove theorems about triangles. Theorems include: a line parallel to one side of a triangle divides the other two proportionally, and conversely; the Pythagorean Theorem proved using triangle similarity.
M.GHS.18	Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures.

Cluster	**Define trigonometric ratios and solve problems involving right triangles.**
M.GHS.19	Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles.
M.GHS.20	Explain and use the relationship between the sine and cosine of complementary angles.
M.GHS.21	Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.

Cluster	**Apply trigonometry to general triangles.**
M.GHS.22	Derive the formula A = 1/2 ab sin(C) for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side.
M.GHS.23	Prove the Laws of Sines and Cosines and use them to solve problems. Instructional Note: With respect to the general case of the Laws of Sines and Cosines, the definitions of sine and cosine must be extended to obtuse angles.

M.GHS.24	Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles. Instructional Note: With respect to the general case of the Laws of Sines and Cosines, the definitions of sine and cosine must be extended to obtuse angles.

Extending to Three Dimensions

Cluster	Explain volume formulas and use them to solve problems.
M.GHS.25	Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone. Use dissection arguments, Cavalieri's principle, and informal limit arguments. Instructional Note: Informal arguments for area and volume formulas can make use of the way in which area and volume scale under similarity transformations: when one figure in the plane results from another by applying a similarity transformation with scale factor k, its area is k^2 times the area of the first. Similarly, volumes of solid figures scale by k^3 under a similarity transformation with scale factor k.
M.GHS.26	Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems. Instructional Note: Informal arguments for area and volume formulas can make use of the way in which area and volume scale under similarity transformations: when one figure in the plane results from another by applying a similarity transformation with scale factor k, its area is k^2 times the area of the first. Similarly, volumes of solid figures scale by k^3 under a similarity transformation with scale factor k.

Cluster	Visualize the relation between two dimensional and three-dimensional objects.
M.GHS.27	Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects.

Cluster	Apply geometric concepts in modeling situations.
M.GHS.28	Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder). Instructional Note: Focus on situations that require relating two- and three-dimensional objects, determining and using volume, and the trigonometry of general triangles.

Connecting Algebra and Geometry Through Coordinates
(This unit has a close connection with the unit, Circles With and Without Coordinates. Reasoning with triangles in this unit is limited to right triangles; e.g., derive the equation for a line through two points using similar right triangles. Relate work on parallel lines to work in High School Algebra I involving systems of equations having no solution or infinitely many solutions. M.GHS.32 provides practice with the distance formula and its connection with the Pythagorean Theorem.)

Cluster	Use coordinates to prove simple geometric theorems algebraically.
M.GHS.29	Use coordinates to prove simple geometric theorems algebraically. (e.g., Prove or disprove that a figure defined by four given points in the coordinate plane is a rectangle; prove or disprove that the point (1, √3) lies on the circle centered at the origin and containing the point (0, 2).
M.GHS.30	Prove the slope criteria for parallel and perpendicular lines and uses them to solve geometric problems. (e.g., Find the equation of a line parallel or perpendicular to a

	given line that passes through a given point.) Instructional Note: Relate work on parallel lines to work in High School Algebra I involving systems of equations having no solution or infinitely many solutions.
M.GHS.31	Find the point on a directed line segment between two given points that partitions the segment in a given ratio.
M.GHS.32	Use coordinates to compute perimeters of polygons and areas of triangles and rectangles, e.g., using the distance formula. This standard provides practice with the distance formula and its connection with the Pythagorean theorem.

Cluster	**Translate between the geometric description and the equation for a conic section.**
M.GHS.33	Derive the equation of a parabola given a focus and directrix. Instructional Note: The directrix should be parallel to a coordinate axis.

Circles With and Without Coordinates

Cluster	**Understand and apply theorems about circles.**
M.GHS.34	Prove that all circles are similar.
M.GHS.35	Identify and describe relationships among inscribed angles, radii, and chords. Include the relationship between central, inscribed, and circumscribed angles; inscribed angles on a diameter are right angles; the radius of a circle is perpendicular to the tangent where the radius intersects the circle.
M.GHS.36	Construct the inscribed and circumscribed circles of a triangle, and prove properties of angles for a quadrilateral inscribed in a circle.
M.GHS.37	Construct a tangent line from a point outside a given circle to the circle.

Cluster	**Find arc lengths and areas of sectors of circles.**
M.GHS.38	Derive using similarity the fact that the length of the arc intercepted by an angle is proportional to the radius, and define the radian measure of the angle as the constant of proportionality; derive the formula for the area of a sector. Instructional Note: Emphasize the similarity of all circles. Reason that by similarity of sectors with the same central angle, arc lengths are proportional to the radius. Use this as a basis for introducing radian as a unit of measure. It is not intended that it be applied to the development of circular trigonometry in this course.

Cluster	**Translate between the geometric description and the equation for a conic section.**
M.GHS.39	Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation.

Cluster	**Use coordinates to prove simple geometric theorems algebraically.**
M.GHS.40	Use coordinates to prove simple geometric theorems algebraically. (e.g., Prove or disprove that a figure defined by four given points in the coordinate plane is a rectangle; prove or disprove that the point (1, $\sqrt{3}$) lies on the circle centered at the origin and containing the point (0, 2).) Instructional Note: Include simple proofs involving circles.

Cluster	**Apply geometric concepts in modeling situations.**

M.GHS.41	Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder). Instructional Note: Focus on situations in which the analysis of circles is required.

Applications of Probability

Cluster	**Understand independence and conditional probability and use them to interpret data.**
M.GHS.42	Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events ("or," "and," "not").
M.GHS.43	Understand that two events A and B are independent if the probability of A and B occurring together is the product of their probabilities, and use this characterization to determine if they are independent.
M.GHS.44	Recognize the conditional probability of A given B as P(A and B)/P(B), and interpret independence of A and B as saying that the conditional probability of A given B is the same as the probability of A, and the conditional probability of B given A is the same as the probability of B. Instructional Note: Build on work with two-way tables from Algebra I to develop understanding of conditional probability and independence.
M.GHS.45	Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities. For example, collect data from a random sample of students in your school on their favorite subject among math, science, and English. Estimate the probability that a randomly selected student from your school will favor science given that the student is in tenth grade. Do the same for other subjects and compare the results. Instructional Note: Build on work with two-way tables from Algebra I to develop understanding of conditional probability and independence.
M.GHS.46	Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations. For example, compare the chance of having lung cancer if you are a smoker with the chance of being a smoker if you have lung cancer.

Cluster	**Use the rules of probability to compute probabilities of compound events in a uniform probability model.**
M.GHS.47	Find the conditional probability of A given B as the fraction of B's outcomes that also belong to A, and interpret the answer in terms of the model.
M.GHS.48	Apply the Addition Rule, P(A or B) = P(A) + P(B) – P(A and B), and interpret the answer in terms of the model.
M.GHS.49	Apply the general Multiplication Rule in a uniform probability model, P(A and B) = P(A)P(B|A) = P(B)P(A|B), and interpret the answer in terms of the model.
M.GHS.50	Use permutations and combinations to compute probabilities of compound events and solve problems.

Cluster	**Use probability to evaluate outcomes of decisions.** Instructional Note: This unit sets the stage for work in Algebra II, where the ideas of statistical inference are introduced. Evaluating the risks associated with conclusions

	drawn from sample data (i.e. incomplete information) requires an understanding of probability concepts.
M.GHS.51	Use probabilities to make fair decisions (e.g., drawing by lots and/or using a random number generator).
M.GHS.52	Analyze decisions and strategies using probability concepts (e.g., product testing, medical testing, and/or pulling a hockey goalie at the end of a game).

Modeling with Geometry

Cluster	**Visualize relationships between two dimensional and three-dimensional objects and apply geometric concepts in modeling situations.**
M.GHS.53	Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).
M.GHS.54	Apply concepts of density based on area and volume in modeling situations (e.g., persons per square mile, BTUs per cubic foot).
M.GHS.55	Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios).

Mathematics - High School Algebra II

All West Virginia teachers are responsible for classroom instruction that integrates content standards and mathematical habits of mind. Students in this course will build on their work with linear, quadratic, and exponential functions and extend their repertoire of functions to include polynomial, rational, and radical functions. (In this course rational functions are limited to those whose numerators are of degree at most 1 and denominators of degree at most 2; radical functions are limited to square roots or cube roots of at most quadratic polynomials.) Students will work closely with the expressions that define the functions, and continue to expand and hone their abilities to model situations and to solve equations, including solving quadratic equations over the set of complex numbers and solving exponential equations using the properties of logarithms. Students will continue developing mathematical proficiency in a developmentally-appropriate progressions of standards. Mathematical habits of mind, which should be integrated in these content areas, include: making sense of problems and persevering in solving them, reasoning abstractly and quantitatively; constructing viable arguments and critiquing the reasoning of others; modeling with mathematics; using appropriate tools strategically; attending to precision, looking for and making use of structure; and looking for and expressing regularity in repeated reasoning. Continuing the skill progressions from previous courses, the following chart represents the mathematical understandings that will be developed:

Polynomial, Rational, and Radical Relationships	Trigonometric Functions
• Derive the formula for the sum of a geometric series, and use the formula to solve problems. (e.g., Calculate mortgage payments.)	• Apply knowledge of trigonometric functions to determine distances in realistic situations. (e.g., Determine heights of inaccessible objects.)
Modeling with Functions	**Inferences and Conclusions from Data**
• Analyze real-world situations using mathematics to understand the situation better and optimize, troubleshoot, or make an informed decision. (e.g., Estimate water and food needs in a disaster area, or use volume formulas and graphs to find an optimal size for an industrial package.) • Analyze decisions and strategies using probability concepts (e.g., product testing, medical testing, pulling a hockey goalie at the end of a game).	• Make inferences and justify conclusions from sample surveys, experiments, and observational studies.

Numbering of Standards

The following Mathematics Standards will be numbered continuously. The following ranges relate to the clusters found within Mathematics:

Polynomial, Rational, and Radical Relationships	
Perform arithmetic operations with complex numbers.	Standards 1-2
Use complex numbers in polynomial identities and equations.	Standards 3-5

Interpret the structure of expressions.	Standard 6-7
Write expressions in equivalent forms to solve problems.	Standard 8
Perform arithmetic operations on polynomials.	Standard 9
Understand the relationship between zeros and factors of polynomials.	Standard 10-11
Use polynomial identities to solve problems.	Standards 12-13
Rewrite rational expressions.	Standards 14-15
Understand solving equations as a process of reasoning and explain the reasoning.	Standard 16
Represent and solve equations and inequalities graphically.	Standard 17
Analyze functions using different representations.	Standard 18
Trigonometric Functions	
Extend the domain of trigonometric functions using the unit circle.	Standards 19-20
Model periodic phenomena with trigonometric functions.	Standard 21
Prove and apply trigonometric identities.	Standard 22
Modeling with Functions	
Create equations that describe numbers or relationships.	Standards 23-26
Interpret functions that arise in applications in terms of a context.	Standards 27-29
Analyze functions using different representations.	Standards 30-32
Build a function that models a relationship between two quantities.	Standard 33
Build new functions from existing functions.	Standards 34-35
Construct and compare linear, quadratic, and exponential models and solve problems.	Standard 36
Inferences and Conclusions from Data	
Summarize, represent, and interpret data on a single count or measurement variable.	Standard 37
Understand and evaluate random processes underlying statistical experiments.	Standard 38-39
Make inferences and justify conclusions from sample surveys, experiments, and observational studies.	Standards 40-43
Use probability to evaluate outcomes of decisions.	Standards 44-45

Polynomial, Rational, and Radical Relationships

Cluster	Perform arithmetic operations with complex numbers.
M.A2HS.1	Know there is a complex number i such that $i^2 = -1$, and every complex number has the form a + bi with a and b real.

M.A2HS.2	Use the relation $i^2 = -1$ and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers.

Cluster	Use complex numbers in polynomial identities and equations.
M.A2HS.3	Solve quadratic equations with real coefficients that have complex solutions. Instructional Note: Limit to polynomials with real coefficients.
M.A2HS.4	Extend polynomial identities to the complex numbers. For example, rewrite $x^2 + 4$ as $(x + 2i)(x – 2i)$. Instructional Note: Limit to polynomials with real coefficients.
M.A2HS.5	Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials. Instructional Note: Limit to polynomials with real coefficients.

Cluster	Interpret the structure of expressions.
M.A2HS.6	Interpret expressions that represent a quantity in terms of its context. a. Interpret parts of an expression, such as terms, factors, and coefficients. b. Interpret complicated expressions by viewing one or more of their parts as a single entity. For example, interpret $P(1 + r)^n$ as the product of P and a factor not depending on P. Instructional Note: Extend to polynomial and rational expressions.
M.A2HS.7	Use the structure of an expression to identify ways to rewrite it. For example, see $x^4 – y^4$ as $(x^2)^2 – (y^2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2 – y^2)(x^2 + y^2)$. Instructional Note: Extend to polynomial and rational expressions.

Cluster	Write expressions in equivalent forms to solve problems.
M.A2HS.8	Derive the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems. For example, calculate mortgage payments. Instructional Note: Consider extending this standard to infinite geometric series in curricular implementations of this course description.

Cluster	Perform arithmetic operations on polynomials.
M.A2HS.9	Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials. Instructional Note: Extend beyond the quadratic polynomials found in Algebra I.

Cluster	Understand the relationship between zeros and factors of polynomials.
M.A2HS.10	Know and apply the Remainder Theorem: For a polynomial $p(x)$ and a number a, the remainder on division by $x – a$ is $p(a)$, so $p(a) = 0$ if and only if $(x – a)$ is a factor of $p(x)$.
M.A2HS.11	Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.

Cluster	Use polynomial identities to solve problems.
M.A2HS.12	Prove polynomial identities and use them to describe numerical relationships. For example, the polynomial identity $(x^2 + y^2)^2 = (x^2 – y^2)^2 + (2xy)^2$ can be used to generate Pythagorean triples. Instructional Note: This cluster has many possibilities for optional enrichment, such as relating the example in M.A2HS.10 to the solution

	of the system $u^2 + v^2 = 1$, $v = t(u+1)$, relating the Pascal triangle property of binomial coefficients to $(x + y)^{n+1} = (x + y)(x + y)^n$, deriving explicit formulas for the coefficients, or proving the binomial theorem by induction.
M.A2HS.13	Know and apply the Binomial Theorem for the expansion of $(x + y)^n$ in powers of x and y for a positive integer n, where x and y are any numbers, with coefficients determined for example by Pascal's Triangle. Instructional Note: This cluster has many possibilities for optional enrichment, such as relating the example in M.A2HS.10 to the solution of the system $u^2 + v^2 = 1$, $v = t(u+1)$, relating the Pascal triangle property of binomial coefficients to $(x + y)^{n+1} = (x + y)(x + y)^n$, deriving explicit formulas for the coefficients, or proving the binomial theorem by induction.

Cluster	**Rewrite rational expressions.**
M.A2HS.14	Rewrite simple rational expressions in different forms; write a(x)/b(x) in the form q(x) + r(x)/b(x), where a(x), b(x), q(x), and r(x) are polynomials with the degree of r(x) less than the degree of b(x), using inspection, long division, or, for the more complicated examples, a computer algebra system. Instructional Note: The limitations on rational functions apply to the rational expressions.
M.A2HS.15	Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions. Instructional Note: This standard requires the general division algorithm for polynomials.

Cluster	**Understand solving equations as a process of reasoning and explain the reasoning.**
M.A2HS.16	Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise. Instructional Note: Extend to simple rational and radical equations.

Cluster	**Represent and solve equations and inequalities graphically.**
M.A2HS.17	Explain why the x-coordinates of the points where the graphs of the equations y = f(x) and y = g(x) intersect are the solutions of the equation f(x) = g(x); find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Instructional Note: Include cases where f(x) and/or g(x) are linear, polynomial, rational, absolute value, exponential, and logarithmic functions. Instructional Note: Include combinations of linear, polynomial, rational, radical, absolute value, and exponential functions.

Cluster	**Analyze functions using different representations.**
M.A2HS.18	Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior. Instructional Note: Relate this standard to the relationship between zeros of quadratic functions and their factored forms.

Trigonometric Functions

Cluster	**Extend the domain of trigonometric functions using the unit circle.**

M.A2HS.19	Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.
M.A2HS.20	Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles

Cluster	**Model periodic phenomena with trigonometric functions.**
M.A2HS.21	Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.

Cluster	Prove and apply trigonometric identities.
M.A2HS.22	Prove the Pythagorean identity $\sin^2(\theta) + \cos^2(\theta) = 1$ and use it to find sin (θ), cos (θ), or tan (θ), given sin (θ), cos (θ), or tan (θ), and the quadrant of the angle. Instructional Note: An Algebra II course with an additional focus on trigonometry could include the standard “Prove the addition and subtraction formulas for sine, cosine, and tangent and use them to solve problems.” This could be limited to acute angles in Algebra II.

Modeling with Functions

Cluster	**Create equations that describe numbers or relationships.**
M.A2HS.23	Create equations and inequalities in one variable and use them to solve problems. Instructional Note: Include equations arising from linear and quadratic functions, and simple rational and exponential functions.
M.A2HS.24	Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales. Instructional Note: While functions will often be linear, exponential, or quadratic the types of problems should draw from more complex situations than those addressed in Algebra I. (e.g., Finding the equation of a line through a given point perpendicular to another line allows one to find the distance from a point to a line).
M.A2HS.25	Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context. (e.g., Represent inequalities describing nutritional and cost constraints on combinations of different foods.) Instructional Note: While functions will often be linear, exponential, or quadratic the types of problems should draw from more complex situations than those addressed in Algebra I. For example, finding the equation of a line through a given point perpendicular to another line allows one to find the distance from a point to a line.
M.A2HS.26	Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. (e.g., Rearrange Ohm’s law V = IR to highlight resistance R.) While functions will often be linear, exponential, or quadratic the types of problems should draw from more complex situations than those addressed in Algebra I. For example, finding the equation of a line through a given point perpendicular to another line allows one to find the distance from a point to a line. This example applies to earlier instances of this standard, not to the current course.

Cluster	**Interpret functions that arise in applications in terms of a context.**
M.A2HS.27	For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity. Instructional Note: Emphasize the selection of a model function based on behavior of data and context.
M.A2HS.28	Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. (e.g., If the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.) Note: Emphasize the selection of a model function based on behavior of data and context.
M.A2HS.29	Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph. Note: Emphasize the selection of a model function based on behavior of data and context.

Cluster	**Analyze functions using different representations.**
M.A2HS.30	Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. a. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions. b. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude. Instructional Note: Focus on applications and how key features relate to characteristics of a situation, making selection of a particular type of function model appropriate.
M.A2HS.31	Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. Instructional Note: Focus on applications and how key features relate to characteristics of a situation, making selection of a particular type of function model appropriate.
M.A2HS.32	Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). (e.g., Given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.) Instructional Note: Focus on applications and how key features relate to characteristics of a situation, making selection of a particular type of function model appropriate.

Cluster	**Build a function that models a relationship between two quantities.**
M.A2HS.33	Write a function that describes a relationship between two quantities. Combine standard function types using arithmetic operations. (e.g., Build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.) Instructional Note: Develop models for more complex or sophisticated situations than in previous courses.

Cluster	Build new functions from existing functions.
M.A2HS.34	Identify the effect on the graph of replacing f(x) by f(x) + k, k f(x), f(kx), and f(x + k) for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them. Instructional Note: Use transformations of functions to find models as students consider increasingly more complex situations. Observe the effect of multiple transformations on a single graph and the common effect of each transformation across function types.
M.A2HS.35	Find inverse functions. Solve an equation of the form f(x) = c for a simple function f that has an inverse and write an expression for the inverse. (e.g., $f(x) = 2x^3$ or f(x) = (x+1)/(x-1) for x ≠ 1.) Instructional Note: Use transformations of functions to find models as students consider increasingly more complex situations. Extend this standard to simple rational, simple radical, and simple exponential functions; connect this standard to M.A2HS.34.

Cluster	Construct and compare linear, quadratic, and exponential models and solve problems.
M.A2HS.36	For exponential models, express as a logarithm the solution to a b^{ct} = d where a, c, and d are numbers and the base b is 2, 10, or e; evaluate the logarithm using technology. Instructional Note: Consider extending this unit to include the relationship between properties of logarithms and properties of exponents, such as the connection between the properties of exponents and the basic logarithm property that log xy = log x +log y.

Inferences and Conclusions from Data

Cluster	Summarize, represent, and interpret data on a single count or measurement variable.
M.A2HS.37	Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets, and tables to estimate areas under the normal curve. Instructional Note: While students may have heard of the normal distribution, it is unlikely that they will have prior experience using it to make specific estimates. Build on students' understanding of data distributions to help them see how the normal distribution uses area to make estimates of frequencies (which can be expressed as probabilities). Emphasize that only some data are well described by a normal distribution.

Cluster	Understand and evaluate random processes underlying statistical experiments.
M.A2HS.38	Understand statistics as a process for making inferences about population parameters based on a random sample from that population. Instructional Note: Include comparing theoretical and empirical results to evaluate the effectiveness of a treatment.
M.A2HS.39	Decide if a specified model is consistent with results from a given data-generating process, e.g., using simulation. (e.g., A model says a spinning coin falls heads up with probability 0.5. Would a result of 5 tails in a row cause you to question the model?)

Cluster	**Make inferences and justify conclusions from sample surveys, experiments, and observational studies.**
M.A2HS.40	Recognize the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each. Instructional Note: In earlier grades, students are introduced to different ways of collecting data and use graphical displays and summary statistics to make comparisons. These ideas are revisited with a focus on how the way in which data is collected determines the scope and nature of the conclusions that can be drawn from that data. The concept of statistical significance is developed informally through simulation as meaning a result that is unlikely to have occurred solely as a result of random selection in sampling or random assignment in an experiment.
M.A2HS.41	Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling. Instructional Note: In earlier grades, students are introduced to different ways of collecting data and use graphical displays and summary statistics to make comparisons. These ideas are revisited with a focus on how the way in which data is collected determines the scope and nature of the conclusions that can be drawn from that data. The concept of statistical significance is developed informally through simulation as meaning a result that is unlikely to have occurred solely as a result of random selection in sampling or random assignment in an experiment. Focus on the variability of results from experiments—that is, focus on statistics as a way of dealing with, not eliminating, inherent randomness.
M.A2HS.42	Use data from a randomized experiment to compare two treatments; use simulations to decide if differences between parameters are significant. Instructional Note: In earlier grades, students are introduced to different ways of collecting data and use graphical displays and summary statistics to make comparisons. These ideas are revisited with a focus on how the way in which data is collected determines the scope and nature of the conclusions that can be drawn from that data. The concept of statistical significance is developed informally through simulation as meaning a result that is unlikely to have occurred solely as a result of random selection in sampling or random assignment in an experiment. Focus on the variability of results from experiments—that is, focus on statistics as a way of dealing with, not eliminating, inherent randomness.
M.A2HS.43	Evaluate reports based on data. Instructional Note: In earlier grades, students are introduced to different ways of collecting data and use graphical displays and summary statistics to make comparisons. These ideas are revisited with a focus on how the way in which data is collected determines the scope and nature of the conclusions that can be drawn from that data. The concept of statistical significance is developed informally through simulation as meaning a result that is unlikely to have occurred solely as a result of random selection in sampling or random assignment in an experiment.

Cluster	**Use probability to evaluate outcomes of decisions.**
M.A2HS.44	Use probabilities to make fair decisions (e.g., drawing by lots or using a random number generator). Instructional Note: Extend to more complex probability models. Include situations such as those involving quality control, or diagnostic tests that yield both false positive and false negative results.

M.A2HS.45	Analyze decisions and strategies using probability concepts (e.g., product testing, medical testing, and/or pulling a hockey goalie at the end of a game). Instructional Note: Extend to more complex probability models. Include situations such as those involving quality control, or diagnostic tests that yield both false positive and false negative results.

FOURTH COURSE OPTIONS

Fourth course options are available to students in either pathway:

- Advanced Mathematical Modeling
- Calculus
- High School Mathematics IV - Trigonometry/Pre-calculus
- STEM Readiness
- Transition Mathematics for Seniors
- AP® Calculus
- AP® Computer Science
- AP® Statistics

Additional course options include dual credit mathematics courses and advanced mathematics courses offered through WV Virtual School. School teams, including counselors, teachers and administrators, should confer with the student and his/her parents to decide what fourth year mathematics course best meets the needs of the student.

Mathematics – Advanced Mathematical Modeling

All West Virginia teachers are responsible for classroom instruction that integrates content standards and mathematical habits of mind. Primary focal points of Advanced Mathematical Modeling include the analysis of information using statistical methods and probability, modeling change and mathematical relationships, mathematical decision making in finance, and spatial and geometric modeling for decision-making. Students will learn to become critical consumers of the quantitative data that surround them every day, knowledgeable decision makers who use logical reasoning and mathematical thinkers who can use their quantitative skills to solve problems related to a wide range of situations. As students solve problems in various applied situations, they will develop critical skills for success in college and careers, including investigation, research, collaboration and both written and oral communication of their work. As students work with these topics, they will rely on mathematical processes, including problem-solving techniques, appropriate mathematical language and communication skills, connections within and outside mathematics and reasoning. Students will use multiple representations, technology, applications and modeling and numerical fluency in problem-solving contexts. Mathematical habits of mind, which should be integrated in these content areas, include: making sense of problems and persevering in solving them, reasoning abstractly and quantitatively; constructing viable arguments and critiquing the reasoning of others; modeling with mathematics; using appropriate tools strategically; attending to precision, looking for and making use of structure; and looking for and expressing regularity in repeated reasoning. Students will continue developing mathematical proficiency in a developmentally-appropriate progressions of standards. Continuing the skill progressions from previous courses, the following chart represents the mathematical understandings that will be developed:

Developing College and Career Skills	Finance
• Develop and apply skills used in college and careers, including reasoning, planning and communication, to make decisions and solve problems in applied situations.	• Create and analyze mathematical models to make decisions related to earning, investing, spending and borrowing money.
Probability	**Statistics**

• Use basic rules of counting and probability to analyze and evaluate risk and return in the context of everyday situations.	• Make decisions based on understanding, analysis and critique of reported statistical information and summaries.
Modeling	**Networks**
• Analyze numerical data in everyday situations using a variety of quantitative measures and numerical processes.	• Use a variety of network models represented graphically to organize data in quantitative situations, make informed decisions, and solve problems.
Social Decision Making	**Geometry**
• Analyze the mathematics behind various methods of ranking and selection and consider the advantages/disadvantages of each method.	• Solve geometric problems involving inaccessible distances. • Use vectors to solve applied problems.

Numbering of Standards

The following Mathematics Standards will be numbered continuously. The following ranges relate to the clusters found within Mathematics:

Developing College and Career Skills	
Math as a language.	Standards 1-2
Tools for problem solving.	Standard 3
Finance	
Understanding financial models.	Standards 4-6
Personal use of finance.	Standards 7-8
Probability	
Analyzing information using probability and counting.	Standards 9-10
Managing uncertainty.	Standards 11-12
Statistics	
Critiquing statistics.	Standards 13-16
Conducting statistical analysis.	Standards 17-21
Communicating statistical information.	Standards 22-23
Modeling	
Managing numerical data.	Standards 24-25
Modeling data and change with functions.	Standards 26-30
Networks	
Networking for decision making.	Standards 31-32
Social Decision Making	
Making decisions using ranking and voting.	Standards 33-34
Geometry	
Concrete geometric representation (physical modeling).	Standards 35-36
Abstract geometric representation (matrix modeling).	Standards 37-38

Developing College and Career Skills

Cluster	Math as a language
M.AMM.1	Demonstrate reasoning skills in developing, explaining and justifying sound mathematical arguments and analyze the soundness of mathematical arguments of others.
M.AMM.2	Communicate with and about mathematics orally and in writing as part of independent and collaborative work, including making accurate and clear presentations of solutions to problems.

Cluster	Tools for problem solving
M.AMM.3	Gather data, conduct investigations and apply mathematical concepts and models to solve problems in mathematics and other disciplines.

Finance

Cluster	Understanding financial models
M.AMM.4	Determine, represent and analyze mathematical models for loan amortization and the effects of different payments and/or finance terms (e.g., Auto, Mortgage, and/or Credit Card).
M.AMM.5	Determine, represent and analyze mathematical models for investments involving simple and compound interest with and without additional deposits. (e.g., Savings accounts, bonds, and/or certificates of deposit.)
M.AMM.6	Determine, represent, and analyze mathematical models for Inflation and the Consumer Price Index using concepts of rate of change and percentage growth.

Cluster	Personal use of finance
M.AMM.7	Research and analyze personal budgets based on given parameters (e.g., Fixed and discretionary expenses, insurance, gross vs. net pay, types of income, wage, salary, commission), career choice, geographic region, retirement and/or investment planning, etc.).
M.AMM.8	Research and analyze taxes including payroll, sales, personal property, real estate and income tax returns.

Probability

Cluster	Analyzing information using probability and counting
M.AMM.9	Use the Fundamental Counting Principle, Permutations and Combinations to determine all possible outcomes for an event; determine probability and odds of a simple event; explain the significance of the Law of Large Numbers.
M.AMM.10	Determine and interpret conditional probabilities and probabilities of compound events by constructing and analyzing representations, including tree diagrams, Venn diagrams, two-way frequency tables and area models, to make decisions in problem situations.

Cluster	Managing uncertainty
M.AMM.11	Use probabilities to make and justify decisions about risks in everyday life.

M.AMM.12	Calculate expected value to analyze mathematical fairness, payoff and risk.

Statistics

Cluster	Critiquing statistics
M.AMM.13	Identify limitations or lack of information in studies reporting statistical information, especially when studies are reported in condensed form.
M.AMM.14	Interpret and compare the results of polls, given a margin of error.
M.AMM.15	Identify uses and misuses of statistical analyses in studies reporting statistics or using statistics to justify particular conclusions, including assertions of cause and effect versus correlation.
M.AMM.16	Describe strengths and weaknesses of sampling techniques, data and graphical displays and interpretations of summary statistics and other results appearing in a study, including reports published in the media.

Cluster	Conducting statistical analysis
M.AMM.17	Identify the population of interest, select an appropriate sampling technique and collect data.
M.AMM.18	Identify the variables to be used in a study.
M.AMM.19	Determine possible sources of statistical bias in a study and how such bias may affect the ability to generalize the results.
M.AMM.20	Create data displays for given data sets to investigate, compare, and estimate center, shape, spread and unusual features.
M.AMM.21	Determine possible sources of variability of data, both those that can be controlled and those that cannot be controlled.

Cluster	Communicating statistical information
M.AMM.22	Report results of statistical studies to a particular audience, including selecting an appropriate presentation format, creating graphical data displays and interpreting results in terms of the question studied.
M.AMM.23	Communicate statistical results in both oral and written formats using appropriate statistical and nontechnical language.

Modeling

Cluster	Managing numerical data
M.AMM.24	Solve problems involving large quantities that are not easily measured.
M.AMM.25	Use arrays to efficiently manage large collections of data and add, subtract, and multiply matrices to solve applied problems.

Cluster	Modeling data and change with functions
M.AMM.26	Determine or analyze an appropriate model for problem situations - including linear, quadratic, power, exponential, logarithmic and logistic functions (e.g., stopping distance, period of a pendulum, population growth, Richter Scale, and/or Fujita Tornado Scale).
M.AMM.27	Determine or analyze an appropriate cyclical model for problem situations that can be modeled with trigonometric functions (e.g., predator-prey models, tide heights,

	diurnal cycle, and/or music).
M.AMM.28	Determine or analyze an appropriate piecewise model for problem situations (e.g., postal rates, phase change graphs, sales tax, and/or utility usage rates).
M.AMM.29	Solve problems using recursion or iteration (e.g., fractals, compound interest, population growth or decline, and/or radioactive decay).
M.AMM.30	Collect numerical bivariate data; use the data to create a scatter plot; determine whether or not a relationship exists; if so, select a function to model the data, justify the selection and use the model to make predictions.

Networks

Cluster	**Networking for decision making**
M.AMM.31	Solve problems involving scheduling or routing situations that can be represented by a vertex-edge graph; find critical paths, Euler paths, Hamiltonian paths, and minimal spanning trees (e.g., Konigsberg bridge problem, mail vs. Fed Ex delivery routes, kolam drawings of India, traveling salesman problem, and/or map coloring).
M.AMM.32	Construct, analyze, and interpret flow charts in order to develop and describe problem solving procedures.

Social Decision Making

Cluster	**Making decisions using ranking and voting**
M.AMM.33	Apply and analyze various ranking algorithms to determine an appropriate method for a given situation (e.g., fair division, apportionment, and/or search engine results).
M.AMM.34	Analyze various voting and selection processes to determine an appropriate method for a given situation (e.g., preferential vs. non-preferential methods, and/or weighted voting).

Geometry

Cluster	**Concrete geometric representation (physical modeling)**
M.AMM.35	Create and use two- and three-dimensional representations of authentic situations using paper techniques or dynamic geometric environments for computer-aided design and other applications.
M.AMM.36	Solve geometric problems involving inaccessible distances.

Cluster	**Abstract geometric representation (matrix modeling)**
M.AMM.37	Use vectors to represent and solve applied problems.
M.AMM.38	Use matrices to represent geometric transformations and solve applied problems.

Calculus Content Standards and Objectives

All West Virginia teachers are responsible for classroom instruction that integrates content standards and objectives and mathematical habits of mind. Students will deepen and extend their understanding of functions, continuity, limits, differentiation, applications of derivatives, integrals, and applications of integration. Students will apply the Rule of Four (Numerical, Analytical, Graphical and Verbal) throughout the course and use available technology to enhance learning. Student will use graphing utilities to investigate concepts and to evaluate derivatives and integrals. Mathematical habits of mind, which should be integrated in these content areas, include: making sense of problems and persevering in solving them, reasoning abstractly and quantitatively; constructing viable arguments and critiquing the reasoning of others; modeling with mathematics; using appropriate tools strategically; attending to precision, looking for and making use of structure; and looking for and expressing regularity in repeated reasoning. Students will continue developing mathematical proficiency in a developmentally-appropriate progressions of standards. Continuing the skill progressions from previous courses, the following chart represents the mathematical understandings that will be developed:

Algebra

- A utility company burns coal to generate electricity. The cost C in dollars of removing p% of the air pollutants emissions is $C = \dfrac{90,000p}{100-p}, \quad 0 \le p < 100.$ Find the cost of removing (a) 10%, (b) 25%, and (c) 75% of the pollutants. Find the limit of C as $p \to 100^{-}$.
- A management company is planning to build a new apartment complex. Knowing the maximum number of apartments the lot can hold and given a function for the maintenance costs, determine the number of apartments that will minimize the maintenance costs.
- The velocity v of the flow of blood at a distance r from the central axis of an artery of radius R is $v = k\left(R^2 - r^2\right)$ where k is the constant of proportionality. Find the average rate of flow of blood along a radius of the artery. (Use 0 and R as the limits of integration.)

Data Analysis and Probability

- The average data entry speeds S (words per minute) of a business student after t weeks of lessons are recorded in the following table.

t	5	10	15	20	25	30
S	28	56	79	90	93	94

Geometry

- The radius of a right circular cylindrical balloon is given by $\sqrt{t+2}$ and its height is $\frac{1}{2}\sqrt{t}$, where t is time in seconds and the dimensions are in inches. Find the rate of change of the volume with respect to time.
- Given 50 meters of framing material, construct a window that will let in the most light if the middle of the window is a rectangle and the top and bottom of the window are semi-circles.
- The graph of f consists of the three line segments joining the points $(0,0),\ (2-2),\ (6,2),$ and $(8,3)$. The function F is defined as follows $F(x) = \int_0^x f(t)\,dt.$ Find the total enclosed areas generated by f and the x-axis. Determine the points of inflection of F on the interval $(0,8)$.

A model for the data is $S = \frac{100t^2}{65+t}, \quad t > 0.$ Do you think that there is a limiting speed? If so, what is the limiting speed? If not, why? • Identify a real life situation that involves quantities that change over time and develop a method to collect and analyze related data. Develop a continuous function to model the data and generalize the results to make a conclusion. • A sheet of typing paper is ruled with parallel lines that are 2 inches apart. A two-inch needle is tossed randomly onto the sheet of paper. The probability that the needle will touch a line is $P = \frac{2}{\pi}\int_0^{\frac{\pi}{2}} \sin\theta \, d\theta$ where θ is the acute angle between the needle and any one of the parallel lines. Find the probability.	

Numbering of Standards

The following Mathematics Standards will be numbered continuously. The following ranges relate to the clusters found within Mathematics:

Algebra	
Understand the key concepts, connections and applications of functions, limits, continuity, derivatives, and integrals represented in multiple ways.	Standards 1-19
Geometry	
Apply the key concepts, connections and applications of limits, continuity, derivatives, and integration for a wide variety of regions.	Standards 20-22
Data Analysis and Probability	
Apply the key concepts and applications of limits, continuity, derivatives, and integration to analyze functions that represent a collection of data.	Standards 23

Algebra

Cluster	Understand the key concepts, connections and applications of functions, limits, continuity, derivatives, and integrals represented in multiple ways.
M.C.1	Use abstract notation to apply properties of algebraic, trigonometric, exponential, logarithmic and composite functions, as well as their inverses, represented graphically,

	numerically, analytically, and verbally; and demonstrate an understanding of the connections among these representations.
M.C.2	Demonstrate a conceptual understanding of the definition of a limit via the analysis of continuous and discontinuous functions represented using multiple representations (e.g. graphs and tables).
M.C.3	Use the properties of limits including addition, product, quotient, composition, and squeeze/sandwich theorem to calculate the various forms of limits: one-sided limits, limits at infinity, infinite limits, limits that do not exist, and special limits such as $\lim_{x\to 0}\frac{\sin x}{x}=1$ and $\lim_{x\to 0}\frac{1-\cos x}{x}=0$.
M.C.4	Apply the definition of continuity to determine where a function is continuous or discontinuous including continuity at a point, continuity over an interval, application of the Intermediate Value Theorem, and graphical interpretation of continuity and discontinuity.
M.C.5	Investigate and apply the definition of the derivative graphically, numerically, and analytically at a point, conceptually interpreting the derivative as an instantaneous rate of change and the slope of the tangent line.
M.C.6	Discriminate between the average rate of change and the instantaneous rate of change using real-world problems.
M.C.7	Recognize when the Extreme Value Theorem indicates that function extrema exist.
M.C.8	Quickly recall and apply rules of differentiation including the constant multiple rule, sum rule, the difference rule, the product rule, the quotient rule, the power rule, and the chain rule as applied to algebraic, trigonometric, exponential, logarithmic, and inverse trigonometric functions using techniques of both explicit and implicit differentiation.
M.C.9	Apply Rolle's Theorem and the Mean Value Theorem to real-world problems.
M.C.10	Construct and use mathematical models to solve optimization, related-rates, velocity, and acceleration problems.
M.C.11	Determine antiderivatives that follow from derivatives of basic functions and apply substitution of variables.
M.C.12	Evaluate definite integrals using basic integration properties such as addition, subtraction, constant multipliers, the power rule, substitution, and change of limits.
M.C.13	Characterize the definite integral as the total change of a function over an interval and use this to solve real-world problems.
M.C.14	Apply the Fundamental Theorem of Calculus to evaluate definite integrals and to formulate a cumulative area function and interpret the function as it relates to the integrand.
M.C.15	Use limits to deduce asymptotic behavior of the graph of a function.
M.C.16	Compare and contrast the limit definition (not delta epsilon) of continuity and the graphical interpretation of the continuity of a function at a point; recognize different types of discontinuities.
M.C.17	Develop tangent lines as best linear approximations to functions near specific points; explain this conceptually; and construct these tangent lines; and apply this concept to Newton's Method.
M.C.18	Investigate and explain the relationships among the graphs of a function, its derivative and its second derivative; construct the graph of a function using the first and second derivatives including extrema, points of inflection, and asymptotic behavior.

M.C.19	Approximate areas under a curve using Riemann sums by applying and comparing left, right, and midpoint methods for a finite number of subintervals.

Geometry

Cluster	**Apply the key concepts, connections and applications of limits, continuity, derivatives, and integration for a wide variety of regions.**
M.C.20	Justify why differentiability implies continuity, and classify functional cases when continuity does not imply differentiability.
M.C.21	Calculate a definite integral using Riemann sums by evaluating an infinite limit of a sum using summation notation and rules for summation.
M.C.22	Use integration to solve problems that involve linear displacement, total distance, position, velocity, acceleration and area between curves by looking at both functions of x and functions of y; utilize units to interpret the physical nature of the calculus process.

Data Analysis and Probability

Cluster	**Apply the key concepts and applications of limits, continuity, derivatives, and integration to analyze functions that represent a collection of data.**
M.C.23	Identify a real life situation that involves quantities that change over time; pose a question; make a hypothesis as to the answer; develop, justify, and implement a method to collect, organize, and analyze related data; extend the nature of collected, discrete data to that of a continuous function that describes the known data set; generalize the results to make a conclusion; compare the hypothesis and the conclusion; present the project numerically, analytically, graphically and verbally using the predictive and analytic tools of calculus.

Mathematics – High School Mathematics IV – Trigonometry/Pre-calculus

All West Virginia teachers are responsible for classroom instruction that integrates content standards and mathematical habits of mind. Students in this course will generalize and abstract learning accumulated through previous courses as the final springboard to calculus. Students will take an extensive look at the relationships among complex numbers, vectors, and matrices. They will build on their understanding of functions, analyze rational functions using an intuitive approach to limits and synthesize functions by considering compositions and inverses. Students will expand their work with trigonometric functions and their inverses and complete the study of the conic sections begun in previous courses. They will enhance their understanding of probability by considering probability distributions and have previous experiences with series augmented. Students will continue developing mathematical proficiency in a developmentally-appropriate progressions of standards. Mathematical habits of mind, which should be integrated in these content areas, include: making sense of problems and persevering in solving them, reasoning abstractly and quantitatively; constructing viable arguments and critiquing the reasoning of others; modeling with mathematics; using appropriate tools strategically; attending to precision, looking for and making use of structure; and looking for and expressing regularity in repeated reasoning. Continuing the skill progressions from previous courses, the following chart represents the mathematical understandings that will be developed:

Building Relationships among Complex Numbers, Vectors, and Matrices	**Analysis and Synthesis of Functions**
• Represent abstract situations involving vectors symbolically.	• Write a function that describes a relationship between two quantities. (e.g., if T(y) is the temperature in the atmosphere as a function of height, and h(t) is the height of a weather balloon as a function of time, then T(h(t)) is the temperature at the location of the weather balloon as a function of time.)
Trigonometric and Inverse Trigonometric Functions of Real Numbers	**Derivations in Analytic Geometry**
• Make sense of the symmetry, periodicity, and special values of trigonometric functions using the unit circle. • Prove trigonometric identities and apply them problem solving situations.	• Make sense of the derivations of the equations of an ellipse and a hyperbola.
Modeling with Probability	**Series and Informal Limits**
• Develop a probability distribution. (e.g., Find the theoretical probability distribution for the number of correct answers obtained by guessing on all five questions of a multiple-choice test where each question has four choices, and find the expected grade under various grading schemes.)	• Apply mathematical induction to prove summation formulas.

Numbering of Standards

The following Mathematics Standards will be numbered continuously. The following ranges relate to the clusters found within Mathematics:

Building Relationships among Complex Numbers, Vectors, and Matrices	
Perform arithmetic operations with complex numbers.	Standard 1
Represent complex numbers and their operations on the complex plane.	Standards 2-4
Represent and model with vector quantities.	Standards 5-7
Perform operations on vectors.	Standards 8-9
Perform operations on matrices and use matrices in applications.	Standards 10-16
Solve systems of equations.	Standards 17-18
Analysis and Synthesis of Functions	
Analyze functions using different representations.	Standard 19
Build a function that models a relationship between two quantities.	Standard 20
Build new functions from existing functions.	Standards 21-22
Trigonometric and Inverse Trigonometric Functions of Real Numbers	
Extend the domain of trigonometric functions using the unit circle.	Standards 23-24
Model periodic phenomena with trigonometric functions.	Standards 25-27
Prove and apply trigonometric identities.	Standard 28
Apply transformations of function to trigonometric functions.	Standard 29
Derivations in Analytic Geometry	
Translate between the geometric description and the equation for a conic section.	Standard 30
Explain volume formulas and use them to solve problems.	Standard 31
Modeling with Probability	
Calculate expected values and use them to solve problems.	Standards 32-35
Use probability to evaluate outcomes of decisions.	Standard 36
Series and Informal Limits	
Use sigma notations to evaluate finite sums.	Standards 37-38
Extend geometric series to infinite geometric series.	Standards 39-40

Building Relationships among Complex Numbers, Vectors, and Matrices

Cluster	Perform arithmetic operations with complex numbers
M.4HSTP.1	Find the conjugate of a complex number; use conjugates to find moduli (magnitude) and quotients of complex numbers. Instructional Note: In Math II students extended the number system to include complex numbers and performed the operations of

	addition, subtraction, and multiplication.
Cluster	**Represent complex numbers and their operations on the complex plane.**
M.4HSTP.2	Represent complex numbers on the complex plane in rectangular and polar form (including real and imaginary numbers), and explain why the rectangular and polar forms of a given complex number represent the same number.
M.4HSTP.3	Represent addition, subtraction, multiplication and conjugation of complex numbers geometrically on the complex plane; use properties of this representation for computation. (e.g., $(-1 + \sqrt{3}\,i)^3 = 8$ because $(-1 + \sqrt{3}\,i)$ has modulus 2 and argument 120°.
M.4HSTP.4	Calculate the distance between numbers in the complex plane as the modulus of the difference and the midpoint of a segment as the average of the numbers at its endpoints.

Cluster	**Represent and model with vector quantities.**
M.4HSTP.5	Recognize vector quantities as having both magnitude and direction. Represent vector quantities by directed line segments and use appropriate symbols for vectors and their magnitudes (e.g., v, |v|, ||v||, v). Instructional Note: This is the student's first experience with vectors. The vectors must be represented both geometrically and in component form with emphasis on vocabulary and symbols.
M.4HSTP.6	Find the components of a vector by subtracting the coordinates of an initial point from the coordinates of a terminal point.
M.4HSTP.7	Solve problems involving velocity and other quantities that can be represented by vectors.

Cluster	**Perform operations on vectors.**
M.4HSTP.8	Add and subtract vectors. a. Add vectors end-to-end, component-wise, and by the parallelogram rule. Understand that the magnitude of a sum of two vectors is typically not the sum of the magnitudes. b. Given two vectors in magnitude and direction form, determine the magnitude and direction of their sum. c. Understand vector subtraction v – w as v + (–w), where –w is the additive inverse of w, with the same magnitude as w and pointing in the opposite direction. Represent vector subtraction graphically by connecting the tips in the appropriate order and perform vector subtraction component-wise.
M.4HSTP.9	Multiply a vector by a scalar. a. Represent scalar multiplication graphically by scaling vectors and possibly reversing their direction; perform scalar multiplication component-wise, e.g., as c(vx, vy) = (cvx, cvy). b. Compute the magnitude of a scalar multiple c**v** using $\lVert c\mathbf{v} \rVert = \lvert c \rvert \cdot \lVert \mathbf{v} \rVert$. Compute the direction of c**v** knowing that when |c|v ≠ 0, the direction of c**v** is either along **v** (for c > 0) or against **v** (for c < 0).

Cluster	**Perform operations on matrices and use matrices in applications.**
M.4HSTP.10	Use matrices to represent and manipulate data (e.g., to represent payoffs or incidence relationships in a network).

M.4HSTP.11	Multiply matrices by scalars to produce new matrices (e.g., as when all of the payoffs in a game are doubled.
M.4HSTP.12	Add, subtract and multiply matrices of appropriate dimensions.
M.4HSTP.13	Understand that, unlike multiplication of numbers, matrix multiplication for square matrices is not a commutative operation, but still satisfies the associative and distributive properties. Instructional Note: This is an opportunity to view the algebraic field properties in a more generic context, particularly noting that matrix multiplication is not commutative.
M.4HSTP.14	Understand that the zero and identity matrices play a role in matrix addition and multiplication similar to the role of 0 and 1 in the real numbers. The determinant of a square matrix is nonzero if and only if the matrix has a multiplicative inverse.
M.4HSTP.15	Multiply a vector (regarded as a matrix with one column) by a matrix of suitable dimensions to produce another vector. Work with matrices as transformations of vectors.
M.4HSTP.16	Work with 2 × 2 matrices as transformations of the plane and interpret the absolute value of the determinant in terms of area. Instructional Note: Matrix multiplication of a 2 x 2 matrix by a vector can be interpreted as transforming points or regions in the plane to different points or regions. In particular a matrix whose determinant is 1 or -1 does not change the area of a region.

Cluster	**Solve systems of equations**
M.4HSTP.17	Represent a system of linear equations as a single matrix equation in a vector variable.
M.4HSTP.18	Find the inverse of a matrix if it exists and use it to solve systems of linear equations (using technology for matrices of dimension 3 × 3 or greater). Instructional Note: Students have earlier solved two linear equations in two variables by algebraic methods.

Analysis and Synthesis of Functions

Cluster	**Analyze functions using different representations.**
M.4HSTP.19	Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior. Instructional Note: This is an extension of graphical analysis from Math III or Algebra II that develops the key features of graphs with the exception of asymptotes. Students examine vertical, horizontal, and oblique asymptotes by considering limits. Students should note the case when the numerator and denominator of a rational function share a common factor. Utilize an informal notion of limit to analyze asymptotes and continuity in rational functions. Although the notion of limit is developed informally, proper notation should be followed.

Cluster	**Build a function that models a relationship between two quantities.**
M.4HSTP.20	Write a function that describes a relationship between two quantities, including composition of functions. For example, if T(y) is the temperature in the atmosphere as a function of height, and h(t) is the height of a weather balloon as a function of time, then T(h(t)) is the temperature at the location of the weather balloon as a function of time.

Cluster	**Build new functions from existing functions.**
M.4HSTP.21	Find inverse functions. Instructional Note: This is an extension of concepts from Math III where the idea of inverse functions was introduced. a. Verify by composition that one function is the inverse of another. b. Read values of an inverse function from a graph or a table, given that the function has an inverse. Instructional Note: Students must realize that inverses created through function composition produce the same graph as reflection about the line y = x.) c. Produce an invertible function from a non-invertible function by restricting the domain. Instructional Note: Systematic procedures must be developed for restricting domains of non-invertible functions so that their inverses exist.)
M.4HSTP.22	Understand the inverse relationship between exponents and logarithms and use this relationship to solve problems involving logarithms and exponents.

Trigonometric and Inverse Trigonometric Functions of Real Numbers

Cluster	**Extend the domain of trigonometric functions using the unit circle.**
M.4HSTP.23	Use special triangles to determine geometrically the values of sine, cosine, tangent for $\pi/3$, $\pi/4$ and $\pi/6$, and use the unit circle to express the values of sine, cosine, and tangent for $\pi-x$, $\pi+x$, and $2\pi-x$ in terms of their values for x, where x is any real number. Instructional Note: Students use the extension of the domain of the trigonometric functions developed in Math III to obtain additional special angles and more general properties of the trigonometric functions.
M.4HSTP.24	Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions.

Cluster	**Model periodic phenomena with trigonometric functions.**
M.4HSTP.25	Understand that restricting a trigonometric function to a domain on which it is always increasing or always decreasing allows its inverse to be constructed.
M.4HSTP.26	Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context. Instructional Note: Students should draw analogies to the work with inverses in the previous unit.
M.4HSTP.27	Solve more general trigonometric equations. (e.g., $2 \sin^2 x + \sin x - 1 = 0$ can be solved using factoring.

Cluster	**Prove and apply trigonometric identities.**
M.4HSTP.28	Prove the addition and subtraction formulas for sine, cosine, and tangent and use them to solve problems.

Cluster	**Apply transformations of function to trigonometric functions.**
M.4HSTP.29	Graph trigonometric functions showing key features, including phase shift. Instructional Note: In Math III, students graphed trigonometric functions showing period, amplitude and vertical shifts.)

Derivations in Analytic Geometry

Cluster	Translate between the geometric description and the equation for a conic section.
M.4HSTP.30	Derive the equations of ellipses and hyperbolas given the foci, using the fact that the sum or difference of distances from the foci is constant. Instructional Note: In Math II students derived the equations of circles and parabolas. These derivations provide meaning to the otherwise arbitrary constants in the formulas.)

Cluster	Explain volume formulas and use them to solve problems.
M.4HSTP.31	Give an informal argument using Cavalieri's principle for the formulas for the volume of a sphere and other solid figures. Instructional Note: Students were introduced to Cavalieri's principle in Math II.

Modeling with Probability

Cluster	Calculate expected values and use them to solve problems.
M.4HSTP.32	Define a random variable for a quantity of interest by assigning a numerical value to each event in a sample space; graph the corresponding probability distribution using the same graphical displays as for data distributions. Instructional Note: Although students are building on their previous experience with probability in middle grades and in Math II and III, this is their first experience with expected value and probability distributions.
M.4HSTP.33	Calculate the expected value of a random variable; interpret it as the mean of the probability distribution.
M.4HSTP.34	Develop a probability distribution for a random variable defined for a sample space in which theoretical probabilities can be calculated; find the expected value. (e.g., Find the theoretical probability distribution for the number of correct answers obtained by guessing on all five questions of a multiple-choice test where each question has four choices, and find the expected grade under various grading schemes.)
M.4HSTP.35	Develop a probability distribution for a random variable defined for a sample space in which probabilities are assigned empirically; find the expected value. For example, find a current data distribution on the number of TV sets per household in the United States, and calculate the expected number of sets per household. How many TV sets would you expect to find in 100 randomly selected households? Instructional Note: It is important that students can interpret the probability of an outcome as the area under a region of a probability distribution graph.

Cluster	Use probability to evaluate outcomes of decisions.
M.4HSTP.36	Weigh the possible outcomes of a decision by assigning probabilities to payoff values and finding expected values. a. Find the expected payoff for a game of chance. (e.g., Find the expected winnings from a state lottery ticket or a game at a fast food restaurant.) b. Evaluate and compare strategies on the basis of expected values. (e.g., Compare a high-deductible versus a low-deductible automobile insurance policy using various, but reasonable, chances of having a minor or a major accident.)

Series and Informal Limits

Cluster	Use sigma notations to evaluate finite sums.
M.4HSTP.37	Develop sigma notation and use it to write series in equivalent form. For example, write $\sum_{i=1}^{n} (3i^2 + 7)$ as $3 \sum_{i=1}^{n} i^2 + 7 \sum_{i=1}^{n} 1$.
M.4HSTP.38	Apply the method of mathematical induction to prove summation formulas. For example, verify that $\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$. Instructional Note: Some students may have encountered induction in Math III in proving the Binomial Expansion Theorem, but for many this is their first experience.

Cluster	Extend geometric series to infinite geometric series.
M.4HSTP.39	Develop intuitively that the sum of an infinite series of positive numbers can converge and derive the formula for the sum of an infinite geometric series. Instructional Note: In Math I, students described geometric sequences with explicit formulas. Finite geometric series were developed in Math III.
M.4HSTP.40	Apply infinite geometric series models. For example, find the area bounded by a Koch curve. Instructional Note: Rely on the intuitive concept of limit developed in unit 2 to justify that a geometric series converges if and only if the ratio is between -1 and 1.

Mathematics – STEM Readiness

All West Virginia teachers are responsible for classroom instruction that integrates content standards and objectives and mathematical habits of mind. This course is designed for students who have completed the Math III (LA) course and subsequently decided they are interested in pursuing a STEM career. It includes standards that would have been covered in Mathematics III (STEM) but not in Mathematics III (LA) (i.e. standards that are marked with a "+"), selected topics from the Mathematics IV course, and topics drawing from standards covered in Mathematics I and Mathematics II as needed for coherence. Mathematical habits of mind, which should be integrated in these content areas, include: making sense of problems and persevering in solving them, reasoning abstractly and quantitatively; constructing viable arguments and critiquing the reasoning of others; modeling with mathematics; using appropriate tools strategically; attending to precision, looking for and making use of structure; and looking for and expressing regularity in repeated reasoning. Students will continue developing mathematical proficiency in a developmentally-appropriate progressions of standards. Continuing the skill progressions from previous courses, the following chart represents the mathematical understandings that will be developed:

Arithmetic and Algebra of Complex Numbers	Polynomial, Rational, and Radical Relationships
• Understand that the arithmetic and algebra of expressions involving rational numbers is governed by the same rules as the arithmetic and algebra of real numbers.	• Derive the formula for the sum of a geometric series, and use the formula to solve problems. (e.g., Calculate mortgage payments.)
Probability for Decisions	**Trigonometry of General Triangles**
• Make inferences and justify conclusions from sample surveys, experiments, and observational studies.	• Apply knowledge of the Law of Sines and the Law of Cosines to determine distances in realistic situations. (e.g., Determine heights of inaccessible objects.)
Functions and Modeling	
• Analyze real-world situations using mathematics to understand the situation better and optimize, troubleshoot, or make an informed decision. (e.g., Estimate water and food needs in a disaster area, or use volume formulas and graphs to find an optimal size for an industrial package.)	

<u>Numbering of Standards</u>

The following Mathematics Standards will be numbered continuously. The following ranges relate to the clusters found within Mathematics:

Arithmetic and Algebra of Complex Numbers	
Perform arithmetic operations with complex numbers.	Standards 1-3
Represent complex numbers and their operations on the complex plane.	Standards 4-6
Use complex numbers in polynomial identities and equations.	Standards 7-9

Polynomial, Rational, and Radical Relationships	
Use polynomial identities to solve problems.	Standard 10
Rewrite rational expressions.	Standard 11
Probability for Decisions	
Use probability to evaluate outcomes of decisions.	Standards 12-13
Trigonometry of General Triangles	
Apply trigonometry to general triangles.	Standards 14-16
Functions and Modeling	
Analyze functions using different representations.	Standards 17-19
Building a function that models a relationship between two quantities.	Standards 20-21
Build new functions from existing functions.	Standards 22-26
Extend the domain of trigonometric functions using the unit circle.	Standards 27-28
Model periodic phenomena using trigonometric functions.	Standards 29-30
Prove and apply trigonometric identities.	Standard 31

Arithmetic and Algebra of Complex Numbers

Cluster	Perform arithmetic operations with complex numbers
M.SRM.1	Know there is a complex number i such that $i^2 = -1$, and every complex number has the form a + bi with a and b real.
M.SRM.2	Use the relation $i^2 = -1$ and the commutative, associative and distributive properties to add, subtract and multiply complex numbers.
M.SRM.3	Find the conjugate of a complex number; use conjugates to find moduli and quotients of complex numbers.

Cluster	Represent complex numbers and their operations on the complex plane
M.SRM.4	Represent complex numbers on the complex plane in rectangular and polar form (including real and imaginary numbers) and explain why the rectangular and polar forms of a given complex number represent the same number.
M.SRM.5	Represent addition, subtraction, multiplication and conjugation of complex numbers geometrically on the complex plane; use properties of this representation for computation. (e.g., $(-1 + \sqrt{3}\, i)^3 = 8$ because $(-1 + \sqrt{3}\, i)$ has modulus 2 and argument 120°.)
M.SRM.6	Calculate the distance between numbers in the complex plane as the modulus of the difference and the midpoint of a segment as the average of the numbers at its endpoints.

Cluster	Use complex numbers in polynomial identities and equations
M.SRM.7	Solve quadratic equations with real coefficients that have complex solutions.
M.SRM.8	Extend polynomial identities to the complex numbers. For example, rewrite $x^2 + 4$ as $(x + 2i)(x - 2i)$.
M.SRM.9	Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials.

Polynomials, Rational, and Radical Relationships

Cluster	Use polynomial identities to solve problems.
M.SRM.10	Know and apply the Binomial Theorem for the expansion of $(x + y)^n$ in powers of x and y for a positive integer n, where x and y are any numbers, with coefficients determined for example by Pascal's Triangle.

Cluster	Rewrite rational expressions
M.SRM.11	Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication and division by a nonzero rational expression; add, subtract, multiply and divide rational expressions.

Probability for Decisions

Cluster	Use probability to evaluate outcomes of decisions.
M.SRM.12	Use probabilities to make fair decisions (e.g. drawing by lot or using a random number generator).
M.SRM.13	Analyze decisions and strategies using probability concepts (e.g., product testing, medical testing, and/or pulling a hockey goalie at the end of a game).

Trigonometry of General Triangles

Cluster	Apply trigonometry to general triangles.
M.SRM.14	Derive the formula A = 1/2 ab sin(C) for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side.
M.SRM.15	Prove the Laws of Sines and Cosines and use them to solve problems.
M.SRM.16	Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles (e.g., surveying problems or resultant forces).

Functions and Modeling

Cluster	Analyze functions using different representations.
M.SRM.17	Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.
M.SRM.18	Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available and showing end behavior.
M.SRM.19	Graph exponential and logarithmic functions, showing intercepts and end behavior and trigonometric functions, showing period, midline, and amplitude.

Cluster	Building a function that models a relationship between two quantities.
M.SRM.20	Write a function that describes a relationship between two quantities.
M.SRM.21	Compose functions. (e.g., If T(y) is the temperature in the atmosphere as a function of height, and h(t) is the height of a weather balloon as a function of time, then T(h(t)) is the temperature at the location of the weather balloon as a function of time.)

Cluster	Build new functions from existing functions.
M.SRM.22	Find inverse functions.
M.SRM.23	Verify by composition that one function is the inverse of another.
M.SRM.24	Read values of an inverse function from a graph or a table, given that the function has an inverse.
M.SRM.25	Produce an invertible function from a non-invertible function by restricting the domain.
M.SRM.26	Understand the inverse relationship between exponents and logarithms and use this relationship to solve problems involving logarithms and exponents.

Cluster	Extend the domain of trigonometric functions using the unit circle.
M.SRM.27	Use special triangles to determine geometrically the values of sine, cosine, tangent for $\pi/3$, $\pi/4$ and $\pi/6$, and use the unit circle to express the values of sine, cosine, and tangent for $\pi-x$, $\pi+x$, and $2\pi-x$ in terms of their values for x, where x is any real number.
M.SRM.28	Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions.

Cluster	Model periodic phenomena using trigonometric functions.
M.SRM.29	Understand that restricting a trigonometric function to a domain on which it is always increasing or always decreasing allows its inverse to be constructed.
M.SRM.30	Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context.

Cluster	Prove and apply trigonometric identities.
M.SRM.31	Prove the addition and subtraction formulas for sine, cosine and tangent and use them to solve problems.

Mathematics – Transition Mathematics for Seniors

All West Virginia teachers are responsible for classroom instruction that integrates content standards and mathematical habits of mind. Transition Mathematics for Seniors prepares students for their entry-level credit-bearing liberal studies mathematics course at the post-secondary level. Students will solidify their quantitative literacy by enhancing numeracy and problem solving skills as they investigate and use the fundamental concepts of algebra, geometry, and introductory trigonometry. Mathematical habits of mind, which should be integrated in these content areas, include: making sense of problems and persevering in solving them, reasoning abstractly and quantitatively; constructing viable arguments and critiquing the reasoning of others; modeling with mathematics; using appropriate tools strategically; attending to precision, looking for and making use of structure; and looking for and expressing regularity in repeated reasoning. Students will continue developing mathematical proficiency in a developmentally-appropriate progressions of standards. Continuing the skill progressions from previous courses, the following chart represents the mathematical understandings that will be developed:

Number and Quantity: The Real Number System The Complex Number System	**Algebra:** Seeing Structure in Expressions Arithmetic with Polynomials and Rational Expressions Creating Equations Reasoning with Equations and Inequalities
• Develop an understanding of basic operations, equivalent representations, and properties of the real and complex number systems.	• Create equations or inequalities that model physical situations. • Solve systems of equations, with an emphasis on efficiency of solution as well as reasonableness of answers, given physical limitations.
Functions: Interpreting Functions Building Functions	**Geometry:** Geometric Measuring and Dimension Expressing Geometric Properties with Equations Modeling with Geometry
• Develop knowledge and understanding of the concept of functions as they use, analyze, represent and interpret functions and their applications.	• Use coordinates and to prove geometric properties algebraically.
Statistics and Probability: Interpreting Categorical and Quantitative Data Making Inferences and Justifying Conclusions	
• Make inferences and justify conclusions from sample surveys, experiments, and observational studies.	

<u>Numbering of Standards</u>

The following Mathematics Standards will be numbered continuously. The following ranges relate to the clusters found within Mathematics:

Number and Quantity – The Real Number System	
Extend the properties of exponents to rational exponents.	Standard 1-2
Number and Quantity – The Complex Number System	
Use complex numbers in polynomial identities and equations.	Standard 3
Algebra – Seeing Structure in Expressions	
Interpret the structure of expressions.	Standard 4
Write expressions in equivalent forms to solve problems.	Standards 5-6
Understand the connections between proportional relationship, lines, and linear equations.	Standards 7-9
Algebra – Arithmetic with Polynomials and Rational Expressions	
Perform arithmetic operations on polynomials.	Standard 10
Algebra – Creating Equations	
Create equations that describe numbers or relationships.	Standards 11-14
Algebra – Reasoning with Equations and Inequalities	
Understand solving equations as a process of reasoning and explain the reasoning.	Standard 15
Solve equations and inequalities in one variable.	Standards 16-18
Solve systems of equations.	Standards 19-21
Represent and solve equations and inequalities graphically.	Standards 22-23
Functions – Interpreting Functions	
Understand the concept of a function and use function notation.	Standard 24
Interpret functions that arise in applications in terms of the context.	Standards 25-28
Analyze functions using different representations.	Standards 29-35
Functions – Building Functions	
Build a function that models a relationship between two quantities.	Standards 36-37
Geometry – Geometric Measuring and Dimension	
Explain volume formulas and use them to solve problems.	Standards 38-39
Geometry – Expressing Geometric Properties with Equations	
Use coordinates to prove simple geometric theorems algebraically.	Standards 40-41
Geometry – Modeling with Geometry	
Apply geometric concepts in modeling situations.	Standard 42
Statistics and Probability – Interpreting Categorical and Quantitative Data	
Summarize, represent, and interpret data on two categorical and quantitative variables.	Standards 43-46
Summarize, represent, and interpret data on a single count or measurement variable.	Standards 47-51

Statistics and Probability – Making Inferences and Justifying Conclusions	
Understand and evaluate random processes underlying statistical experiments.	Standard 52

Number and Quantity – The Real Number System

Cluster	Extend the properties of exponents to rational exponents.
M.TMS.1	Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.
M.TMS.2	Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.

Number and Quantity – The Complex Number System

Cluster	Use complex numbers in polynomial identities and equations.
M.TMS.3	Solve quadratic equations with real coefficients that have complex solutions.

Algebra – Seeing Structure in Expressions

Cluster	Interpret the structure of expressions.
M.TMS.4	Use the structure of an expression to identify ways to rewrite it. For example, see $x^4 – y^4$ as $(x^2)^2 – (y^2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2 – y^2) (x^2 + y^2)$.

Cluster	Write expressions in equivalent forms to solve problems.
M.TMS.5	Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. a. Factor a quadratic expression to reveal the zeros of the function it defines. b. Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines.
M.TMS.6	Derive the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems.

Cluster	Understand the connections between proportional relationship, lines, and linear equations.
M.TMS.7	Graph proportional relationships, interpreting the unit rates as the slope of the graph. Compare two different proportional relationships represented in different ways. For example, compare a distance-time graph to a distance-time equation to determine which of two moving objects has greater speed.
M.TMS.8	Use similar triangles to explain why the slope m is the same between any two distinct points on a non-vertical line in the coordinate plan; derive the equation y = mx for a line through the origin and the equation y = mx + b for a line intercepting the vertical axis at b.
M.MTS.9	Solve linear equations in one variable.

Algebra – Arithmetic with Polynomials and Rational Expressions

Cluster	**Perform arithmetic operations on polynomials.**
M.TMS.10	Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract and multiply polynomials.

Algebra – Creating Equations

Cluster	**Create equations that describe numbers or relationships.**
M.TMS.11	Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions and simple rational and exponential functions.
M.TMS.12	Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.
M.TMS.13	Represent constraints by equations or inequalities and by systems of equations and/or inequalities and interpret solutions as viable or nonviable options in a modeling context. For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.
M.TMS.14	Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations.

Algebra – Reasoning with Equations and Inequalities

Cluster	**Understand solving equations as a process of reasoning and explain the reasoning.**
M.TMS.15	Solve simple rational and radical equations in one variable and give examples showing how extraneous solutions may arise.

Cluster	**Solve equations and inequalities in one variable.**
M.TMS.16	Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.
M.TMS.17	Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.
M.TMS.18	Solve quadratic equations in one variable. Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x – p)^2 = q$ that has the same solutions. Derive the quadratic formula from this form. Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as a ± bi for real numbers a and b.

Cluster	**Solve systems of equations.**
M.TMS.19	Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions.
M.TMS.20	Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically.

M.TMS.21	Explain why the x-coordinates of the points where the graphs of the equation y = f(x) and y = g(x) intersect are the solution of the equation f(x) = g (x); find the solution approximately (e.g., using technology to graph the functions, make tables of values or find successive approximations).

Cluster	**Represent and solve equations and inequalities graphically.**
M.TMS.22	Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.
M.TMS.23	Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality) and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.

Functions – Interpreting Functions

Cluster	**Understand the concept of a function and use function notation.**
M.TMS.24	Understand a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then f(x) denotes the output of f corresponding to the input x. The graph of f is the graph of the equation y = f(x).

Cluster	**Interpret functions that arise in applications in terms of the context.**
M.TMS.25	Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.
M.TMS.26	Interpret the parameters in a linear or exponential function in terms of a context.
M.TMS.27	For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.
M.TMS.28	Distinguish between situations that can be modeled with linear functions and with exponential functions.

Cluster	**Analyze functions using different representations.**
M.TMS.29	Interpret the equation y = mx + b as defining a linear function, whose graph is a straight line, give examples of functions that are not linear.
M.TMS.30	Describe qualitatively the functional relationship between two quantities by analyzing a graph.
M.TMS.31	Identify the effect on the graph of replacing f(x) by f(x) + k, k f(x), f(kx), and f(x + k) for specific values of k (both positive and negative); find the value of k given the graphs.
M.TMS.32	Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. a. Graph linear and quadratic functions and show intercepts, maxima, and minima. b. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.

M.TMS.33	Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasingly linearly, quadratically, or (more generally) as a polynomial function.
M.TMS.34	Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.
M.TMS.35	Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).

Functions - Building Functions

Cluster	**Build a function that models a relationship between two quantities.**
M.TMS.36	Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).
M.TMS.37	Write a function that describes a relationship between two quantities. a. Combine standard function types using arithmetic operations. For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model. b. Compose functions. For example, if T(y) is the temperature in the atmosphere as a function of height, and h(t) is the height of a weather balloon as a function of time, then T(h(t)) is the temperature at the location of the weather balloon as a function of time.

Geometry – Geometric Measuring and Dimension

Cluster	**Explain volume formulas and use them to solve problems.**
M.TMS.38	Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone. Use dissection arguments, Cavalieri's principle, and informal limit arguments.
M.TMS.39	Give an informal argument using Cavalieri's principle for the formulas for the volume of a sphere and other solid figures.

Geometry – Expressing Geometric Properties with Equations

Cluster	**Use coordinates to prove simple geometric theorems algebraically**
M.TMS.40	Use coordinates to prove simple geometric theorems algebraically. For example, prove or disprove that a figure defined by four given points in the coordinate plane is a rectangle; prove or disprove that the point (1, √3) lies on the circle centered at the origin and containing the point (0, 2).
M.TMS.41	Use coordinates to compute perimeters of polygons and areas of triangles and rectangles, (e.g., using the distance formula).

Geometry – Modeling with Geometry

Cluster	Apply geometric concepts in modeling situations.
M.TMS.42	Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with topographic grid systems based on ratios).

Statistics and Probability - Interpreting Categorical & Quantitative Data

Cluster	Summarize, represent, and interpret data on two categorical and quantitative variables.
M.TMS.43	Represent data on two quantitative variables on a scatter plot, and describe how the variables are related. Interpret linear models.
M.TMS.44	Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data.
M.TMS.45	Know that straight lines are widely used to model relationships between two quantitative variables. For scatter plots that suggest a linear association, informally fit a straight line, and informally assess the model fit by judging the closeness of the data points to the line.
M.TMS.46	Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal, and conditional relative frequencies). Recognize possible associations and trends in the data.

Cluster	Summarize, represent, and interpret data on a single count or measurement variable.
M.TMS.47	Represent data with plots on the real number line (dot plots, histograms, and box plots).
M.TMS.48	Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets.
M.TMS.49	Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers).
M.TMS.50	Computer (using technology) and interpret the correlation coefficient of a linear fit.
M.TMS.51	Distinguish between correlation and causation.

Statistics and Probability - Interpreting Categorical & Quantitative Data

Cluster	Understand and evaluate random processes underlying statistical experiments
M.TMS.52	Understand statistics as a process for making inferences about population parameters based on a random sample from that population.